

Questions for discussion, week 35

August 25, 2020

Exercises

Exercise 1. Which of the following expressions might lead to large round-off error for certain choices of x , when evaluated on a computer.

- a) $|x| + x^2$
- b) $1/3 - \sin(x)$
- c) $x + \frac{1}{x}$
- d) $\frac{1}{\sqrt{x^2+1}-x}$

Exercise 2. Below is a list computational problems. Find examples of problems where it might be difficult to compute an accurate approximation on a computer.

- a) Evaluate a real valued function.
- b) Solution to a differential equation using Eulers method
- c) Finding the n 'th iterate in a difference equation (i.e. $x_{k+2} = ax_{k+1} + bx_k$)

Exercise 3. Next we discuss condition numbers.

- a) Recall what a condition number is and try to explain in words what it is trying to say. Why do we care about condition numbers?
- b) Let $A \in \mathbb{C}^{n \times n}$ be non-singular, and let $\|\cdot\|$ be a matrix norm. Recall that the condition number of the matrix A is $K(A) = \|A\| \times \|A^{-1}\|$. Is the choice of norm important here?

Exercise 4. We are given distinct points $x_0, x_1, x_2, x_3 \in \mathbb{R}$ and data $f(x_0), f(x_1), f(x_2), f(x_3) \in \mathbb{R}$.

- a) Write down the Lagrange polynomial for the polynomial interpolant.
- b) A polynomial inpterpolant on the Newton form could look like this

$$p(x) = c_0 + c_1(x - x_0) + c_2(x - x_1)(x - x_0) + c_3(x - x_2)(x - x_1)(x - x_0)$$

for appropriately chosen coefficients $c_0, c_1, c_2, c_3 \in \mathbb{R}$. Is there any advantage/disadvantage of choosing the Newton interpolant instead of the Lagrange interpolant?