Questions for discussion, week 36

September 2, 2020

Exercises

Exercise 1. We are given distinct points $x_0, x_1, x_2, x_3 \in \mathbb{R}$ and data $f(x_0), f(x_1), f(x_2), f(x_3) \in \mathbb{R}$.

a) Write down the Lagrange polynomial for the polynomial interpolant.

b) A polynomial inpterpolant on the Newton form could look like this

$$p(x) = c_0 + c_1(x - x_0) + c_2(x - x_1)(x - x_0) + c_3(x - x_2)(x - x_1)(x - x_0)$$

for appropriately chosen coefficients $c_0, c_1, c_2, c_3 \in \mathbb{R}$. Is there any advantage/disadvantage of choosing the Newton interpolant instead of the Lagrange interpolant?

Exercise 2. We are given distinct points $x_0, x_1, x_2 \in \mathbb{R}$ and data $f(x_0), f(x_1), f(x_2) \in \mathbb{R}$.

a) Is the interpolating polynomial of degree 2 unique? Why/Why not?

b) Can we find an interpolating polynomial of degree 3?

Exercise 3. We discuss linear independence.

a) Let $v_1, v_2, v_3 \in \mathbb{R}^4$ be vectors. What does it mean that v_1, v_2 and v_3 are linearly independent?

b) Consider the polynomials $1, x$ and x^2. What does it mean for these to be linearly independent?

Exercise 4. The book considers least squares polynomial fitting for orthogonal polynomial bases. Do we obtain the same approximation with non-orthogonal bases?