Questions for discussion, week 37

September 8, 2020

Exercises

Exercise 1. (**Convexity**) Let $c^{(1)}, c^{(2)} \in \mathbb{R}^n$ be distinct points.

a) Describe with words what the set $\{tc^{(1)} + (1-t)c^{(2)} : t \in [0,1]\}$ looks like.

b) Let $t_0, t_1 \in \mathbb{R}$ with $t_0 < t_1$. What can you say about

$$\frac{t_1 - t}{t_1 - t_0} c^{(1)} + \frac{t - t_0}{t_1 - t_0} c^{(2)}, \quad \text{for } t \in [t_0, t_1].$$

c) Use what you learn in (b) to deduce that if $p \in \mathbb{P}_n$ interpolates f in $x_0, \ldots, x_n$, and $q \in \mathbb{P}_n$ interpolates f in $x_1, \ldots, x_{n+1}$, then

$$r(x) = \frac{x_{n+1} - x}{x_{n+1} - x_0} p(x) + \frac{x - x_0}{x_{n+1} - x_0} q(x)$$

interpolates f in $x_0, \ldots, x_{n+1}$.

Exercise 2. You are given a dataset $\{(x_i, f(x_i))\}_{i=1}^n \subset \mathbb{R} \times \mathbb{R}$.

a) Recall what properties the spline interpolant to this dataset would have.

b) When would you use splines to fit this dataset?

c) When would you use polynomial to fit this dataset?

Exercise 3. Recall that a spline of degree k with knots $x_0, \ldots, x_n$, can be written as

$$s(x) = \sum_{i=0}^{k} c_i x^i + \sum_{j=1}^{n-1} d_j (x - x_j)_+^k,$$

where we use the notation

$$(x - x_j)_+ = \begin{cases} x - x_j & \text{for } x \geq x_j \\ 0 & \text{otherwise} \end{cases}.$$

a) Show that $(x - x_j)_+^k$ is $k-1$ times continuously differentiable.

b) Why is splines of degree k with fixed knots $x_0, \ldots, x_n$ a linear space (vector space)?

c) What is the dimension of this linear space?