# UNIVERSITY OF OSLO

## Faculty of mathematics and natural sciences

Exam in: MAT3420 — Quantum computing

Day of examination: Monday, June 7, 2021

Examination hours: 9:00 – 13:00

This problem set consists of 2 pages.

Appendices: None

Permitted aids: Any

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

All subproblems (1a, 1b,...) carry the same weight.

## Problem 1

### 1a

Describe how one represents the pure states of a one-qubit system on the Bloch sphere. Draw a picture showing the images of

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \qquad \text{and} \qquad -|0\rangle.$$

### 1b

Prove the formula

$$e^{i\theta\vec{a}\cdot\vec{\sigma}} = (\cos\theta)I + i(\sin\theta)\vec{a}\cdot\vec{\sigma}$$

for all unit vectors $\vec{a} \in \mathbb{R}^3$ and $\theta \in \mathbb{R}$.

### 1c

Describe in words the action of the unitaries $e^{i\theta\vec{a}\cdot\vec{\sigma}}$ on the pure states in the Bloch coordinates. You don't have to justify your answer.

## Problem 2

### 2a

Give the definition of the Schmidt number of a pure state.

### 2b

Compute the Schmidt number of

$$\frac{1}{2}(|0\rangle \otimes |0\rangle + |0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle).$$

*(Continued on page 2.)*

# Problem 3

## 3a

Find the continued fraction expansion of $\frac{23}{16}$.

## 3b

Find all rational numbers $\frac{p}{q}$ satisfying

$$\left|\frac{23}{16} - \frac{p}{q}\right| \le \frac{1}{2q^2}.$$

# Problem 4

Assume $A, B, C, U$ are one-qubit unitary gates satisfying $ABC = I$ and $AXBXC = U$, where $X = \sigma_x$ is the NOT gate. Consider the control-$U$ gate $\Lambda(U)$, so

$$\Lambda(U)(|a\rangle \otimes |b\rangle) = |a\rangle \otimes U^a|b\rangle \quad \text{for all} \quad a, b \in \{0, 1\}.$$

Draw a quantum circuit expressing $\Lambda(U)$ in terms of $A, B, C, X$ and the CNOT gates.

# Problem 5

## 5a

Describe all separable pure states $|\phi\rangle \otimes |\psi\rangle$ of a 2-qubit system such that

$$\text{CNOT}(|\phi\rangle \otimes |\psi\rangle)$$

is again separable, that is, it has the form $|\phi'\rangle \otimes |\psi'\rangle$.

## 5b

Assume we are given a quantum circuit on $k$ qubits, with input state $|0\dots0\rangle$, consisisting of $n$ gates from our standard universal gate set $\{H, T^{\pm 1}, \text{CNOT}\}$ followed by a final measurement of all the qubits. Assume it is known that at every step of the computation the state we get is separable, that is, it is of the form

$$|\phi_1\rangle \otimes \cdots \otimes |\phi_k\rangle.$$

Argue, without going into too many details, that such a quantum computation can be efficiently simulated on a classical computer. More precisely, show that, assuming we can do exact arithmetic operations with real numbers, we need not more than $C_k n$ such operations, for some constant $C_k$ depending on $k$, to compute the probabilities of all possible outcomes $a_1 \dots a_k$ of the quantum computation.

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