MAT3420 2024, Exercises week 7, to be discussed Friday 16 February

Exercise 1. Consider a 2-qubit state (a unit vector in a composite system $\mathbb{C}^2 \otimes \mathbb{C}^2$) given in the form

$$|\varphi\rangle = \gamma_{00}\,|00\rangle + \gamma_{01}\,|01\rangle + \gamma_{10}\,|10\rangle + \gamma_{11}\,|11\rangle$$

for complex scalars $\gamma_{00}, \gamma_{01}, \gamma_{10}, \gamma_{11}$ satisfying $|\gamma_{00}|^2 + |\gamma_{01}|^2 + |\gamma_{10}|^2 + |\gamma_{11}|^2 = 1$. Prove that $|\varphi\rangle$ is a product state in the form

$$(\alpha_0\,|0\rangle + \alpha_1\,|1\rangle) \otimes (\beta_0\,|0\rangle + \beta_1\,|1\rangle)$$

if and only if $\gamma_{00}\gamma_{11} = \gamma_{01}\gamma_{10}$. Hint: to prove the condition is sufficient, try to express the γ's as products of α_j's and β_k's for suitable choices, and use that you may multiply a complex number by $e^{i\theta}$ without changing its modulus, where i is the imaginary complex number in $\mathbb{C}$ and θ is an angle in $[0, 2\pi)$.

Exercise 2. Compute the reduced density operator ρ^B for the Bell state in equation (3.5.20) in [1].

Exercise 3. Exercise 3.5.4 (a) in [1].

Exercise 4. Exercise 4.2.1 in [1].

Exercise 5. Exercise 4.2.4 in [1].

Exercise 6. Exercise 4.2.5 in [1].

References

[1] P. Kaye, R. Laflamme and M. Mosca, An Introduction to Quantum Computing, Oxford University Press, 2007.