



UiO : **University of Oslo**

Workshop on Probability and Combinatorics in Board Games and Card Games

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My background in USA and Singapore

- ▶ Cand. mag., University of Oslo.
- ▶ Ph.D., University of California, Berkeley.
- ▶ 1989–2011, Department of Mathematics, National University of Singapore.
- ▶ Introduced two “General Education Modules”: “Heavenly Mathematics and Cultural Astronomy” and “Mathematics in Art and Architecture”.
- ▶ Vice president, Singapore Mathematical Society.
- ▶ Chair of organizing committee, Singapore Mathematical Olympiad.
- ▶ Reviewer of textbooks, Ministry of Education, Singapore.

My background in Norway

- ▶ Since 2011, joint appointment, Dept. of Teacher Education and Dept. of Mathematics, Univ. of Oslo.
- ▶ Introduced a new course MAT4010, “School Mathematics from an Advanced Viewpoint”.
- ▶ Teach a course every summer in Singapore at a Residential College in University Town.
- ▶ What I do is between mathematics and mathematics education. I call it educational mathematics.

I'm a Game Geek

- ▶ I have always loved board games and card games, and I currently own about 600.



Game Design and Mathematics

- ▶ My favorite designer is Reiner Knizia, who has a Ph.D. in mathematics from the University of Ulm.
(Charakterisierung von mehrdimensionalen Perron-Integralen.)
- ▶ I have made a list on Board Game Geek called Designers with a Background in STEM (Science, Technology, Engineering and Mathematics)

Topics today

- ▶ Scientific and cultural aspects of games.
- ▶ Games that I have used in my children's math classes.

Scientific and cultural aspects of games: Cards

- ▶ Some conservative Jews, Christians and Muslims object to the face cards. “Thou shalt not make unto thee any graven image.”
- ▶ In 1906, Parker Brothers introduced Rook cards. Popular among Mennonites (Anabaptists) and Adventists.
- ▶ The suits are ranked differently in different games. In Bridge it is ♠♥♦♣, in the Chinese game Big Two (Dà lǎo èr, 大老二) it is ♠♥♣♦, in the German game Skat it is ♣♠♥♦ and in Poker there is no ordering.

Scientific and cultural aspects of games: Game table

- ▶ The players in the Chinese game Mahjong are called East (東/东), South (南), West (西) and North (北), but unlike on a regular map or at a Bridge table, they are placed in counter-clockwise order.

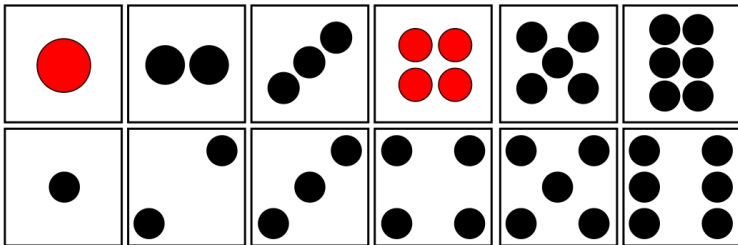


Scientific and cultural aspects of games: Game table 2

- ▶ The Chinese emperor was referred to as the Son of Heaven, and ruled by the Mandate of Heaven. The Mahjong table is supposed to be a celestial map, not a terrestrial map.
- ▶ Many Chinese card games, like Big Two, (Dà lǎo èr, 大老二) are dealt and played counter-clockwise.

Scientific and cultural aspects of games: Dice

- ▶ On a regular D6, the sum of opposite faces equals 7. However, this does not determine the layout of the die completely.
- ▶ On a Western die, the 1, 2 and 3 faces are normally oriented counter-clockwise (right-handed), while Chinese dice are normally left-handed.
- ▶ In addition, Chinese dice normally have a big 1 pip, red 1 and 4 pips, the pips are tightly spaced and the 2 pips are not placed along the diagonal.



Educational Games

- ▶ What is the problem with American football?
- ▶ They don't use their feet and they don't use a ball.
- ▶ That is how I feel about educational games.
- ▶ Many of them are not really educational, but just simple quiz drills.
- ▶ Many of them are not really interesting as games, but just simple roll and move activities with few meaningful decisions.

Do Games Help You Learn Mathematics?

- ▶ Most of the games I own involve one or more of the following components:
 - ▶ Arithmetic
 - ▶ Strategic planning
 - ▶ Probability estimates
 - ▶ Deduction
 - ▶ Logic
 - ▶ Geometric visualization
- ▶ However, does that make them good games for the classroom?
- ▶ Teachers are pressed for time. They need to know that the games I propose will help them reach certain learning outcomes.
- ▶ I compare it to Sudoku. Some studies seem to indicate that Sudoku might help against dementia, but will it help students learn mathematics?

Games that you can use in class

- ▶ Looking for games with the following properties.
- ▶ Clear mathematical content.
- ▶ Simple rules.
- ▶ Inexpensive, available for free online, playable with standard components, or (barely) legally available as print and play.
- ▶ Fun!

Yes, Some Games Can Help You Learn Mathematics

- ▶ I have found some games that I believe are good educational games, and that are practical to use in the classroom. Some of the ones I have used are
 - ▶ Qwixx by Steffen Benndorf (2012), 2013 Spiel des Jahres nominee. (Sum of two dice)
 - ▶ Liar's Dice (also known as Dudo, Bluff or Perudo) by Richard Borg (1987), 1993 Spiel des Jahres winner. (Probability estimates based on counting)
 - ▶ Can't Stop by Sid Sackson (1980), available for free online as Roll or Don't (www.rollordont.com). (Sum of two dice, stopping time)
 - ▶ Reiner Knizia's Decathlon, Print 'n Play available online. (Dice probabilities)
 - ▶ Diamant (also known as Incan Gold) by Bruno Faidutti & Alan R. Moon (2005), 2005 Spiel des Jahres recommended. (Probability estimates based on counting)

Two Dice Discussion 1

- a. Before we start playing, I want to discuss simple dice probabilities. This is from a test that my children had at school.
- b. A. How many outcomes can you get when you roll two regular dice?
- c. B. Which of these outcomes is most likely to occur when you roll two dice?
 - ▶ 1, 6,
 - ▶ 6, 6,
 - ▶ They are equally likely.
- ▶ The answers are given as 36 and 1, 6.
- ▶ Discuss: What is the problem with these answers?

Two Dice Discussion 2

- ▶ In a) they mean ordered, in b) they mean unordered. Both interpretations are possible, but you need to be consistent.
- ▶ The question is whether you can differentiate between the dice.
- ▶ Assume that one die is red and the other is blue. Then there are 36 possible outcomes.

	R-1	R-2	R-3	R-4	R-5	R-6
B-1						
B-2						
B-3						
B-4						
B-5						
B-6						

Two Dice Discussion 3

- ▶ How many different outcomes are there if the dice are identical and are thrown at the same time in a dice cup?
- ▶ There are therefore $15 + 6 = 21$ outcomes. We have a non-uniform probability model, where 15 outcomes (those above the diagonal) have a probability of $1/18$ and 6 outcomes (those on the diagonal) have a probability of $1/36$.

	1	2	3	4	5	6
1	X	XX	XX	XX	XX	XX
2		X	XX	XX	XX	XX
3			X	XX	XX	XX
4				X	XX	XX
5					X	XX
6						X

Sum of Two Dice 1

- ▶ What is the most likely sum if you roll two dice?
- ▶ Pretend that one cube is red and the other is blue.

	R-1	R-2	R-3	R-4	R-5	R-6
B-1	2	3	4	5	6	7
B-2	3	4	5	6	7	8
B-3	4	5	6	7	8	9
B-4	5	6	7	8	9	10
B-5	6	7	8	9	10	11
B-6	7	8	9	10	11	12

- ▶ The probability of getting 7 is $6/36 = 1/6$.
- ▶ If the dice are identical, we have a non-uniform probability model. The probability of getting 7 is $3 \times 1/18 = 1/6$.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2		4	5	6	7	8
3			6	7	8	9
4				8	9	10
5					10	11
6						12

Sum of Two Dice 2

- ▶ This is important in many board games, for example in Settlers of Catan and Monopoly.

STARTING MAP FOR BEGINNERS

To make it as easy as possible for you to get started with *Catan*, we use an award-winning rules system, which consists of 3 parts—the *Overview*, the *Game Rules*, and the *Almanac*.

If you've never played *Catan*, please read the *game Overview* first—it's on the back cover of this booklet. Next, read the *Game Rules* and start to play. And finally, if you have questions during the game, please consult the *Almanac* (it begins on page 6).

RESOURCE PRODUCTION

Begin the game with the resource cards produced by the settlements marked with white stars. See ☆

ODDS FOR DICE ROLLS

2 & 12 =	3%
3 & 11 =	6%
4 & 10 =	8%
5 & 9 =	11%
6 & 8 =	14%
7 =	17%

- ▶ We will now play Qwixx.

Liar's Dice

- ▶ We will now play Liar's Dice.

Can't Stop

- ▶ We will now play Can't Stop, using the web page `www.rollordont.com/`.

Thanks!