School Mathematics from an Advanced Viewpoint Pretest

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Discuss these questions in your group. Afterwards I will ask you if you think the questions were easy or hard.

1 Warmup

1. How can you explain to a 12-year old that $10 : 1/2 = 20$?
2. Is $0.\overline{9} = 1$?
3. What can you say about the sum and product of the roots of a quadratic equation $ax^2 + bx + c = 0$?
4. Why is it not always a good idea to say that the slope of a linear function is how much y increases when x increases by 1?
5. What is a rational function?
6. What do we call a quadrilateral with four equal sides?
7. Why does GeoGebra by default give equations for a line in the plane in the form $ax + by = c$ instead of $y = ax + b$?
8. How can you explain to a 12-year old that the area of a circle of radius r is πr^2?
9. Why and when do we use radians?
10. Is the function $f(x) = 1/x$ continuous?
11. Give an example of a function where $f'(0) = 0$, but where 0 is not an extremum.

12. Give an example of a function where $f''(0) = 0$, but where 0 is not a point of inflection.

13. The Fundamental Theorem of Calculus says that for a differentiable function $f(x)$, we can write $f(x) = \int_a^x f'(t)dt$. Why is this fundamental?

14. Why should you write $f(x) = \int_a^x f'(t)dt$ instead of $f(x) = \int_a^x f'(x)dx$?

2 Harder questions

1. Give an example of a function that has a tangent at a point, but which is not differentiable there.

2. Why is the derivative of the area of a circle of radius r, $A(r) = \pi r^2$, equal to the circumference, $C(r) = 2\pi r$?

3. Can you say something that can make students remember the formulas for the volume of the sphere of radius r, $V(r) = (4/3)\pi r^3$, and the surface area of a sphere, $4\pi r^2$?

4. Give an example of two triangles, ΔABC and $\Delta A'B'C'$, where $AB = A'B'$, $BC = B'$ and $\angle A = \angle A'$, but where the triangles are not congruent.

3 Much harder questions!

1. Which fractions a/b with $a, b \in \mathbb{Z}$ have finite decimal expansion?

2. Why is a geometric series called geometric?

3. If $f(x)$ is differentiable everywhere, $f(x) > 0$ for $x \neq 0$ and $f(0) = 0$, does it follow that $f'(x)$ changes sign at 0?