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Solid Geometry

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Parametric equations 1

- ▶ We use two types of equations in solid geometry: Parametric equations and Cartesian (or coordinate) equations.
- ▶ A parametric equation of a line in \mathbb{R}^n is of the form $\vec{X} = \vec{P} + t\vec{V}$, where $\vec{X} = \{x_1, \dots, x_n\}$ is an arbitrary point on the line, \vec{P} is a given point on the line, and \vec{V} is a direction vector of the line.
- ▶ A parametric equation of a plane in \mathbb{R}^n is of the form $\vec{X} = \vec{P} + s\vec{U} + t\vec{V}$, where $\vec{X} = \{x_1, \dots, x_n\}$ is an arbitrary point on the plane, \vec{P} is a given point on the line, and \vec{U} and \vec{V} are spanning vectors of the plane.

Parametric equations 2

- ▶ Notice that we only need one equation to describe either of these objects. However, if we write the equations componentwise, we will get n equations. For example, we can write the equation for a line in \mathbb{R}^3 as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + t \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

or as

$$x = p_1 + tv_1$$

$$y = p_2 + tv_2$$

$$z = p_3 + tv_3.$$

Cartesian equations 1

- ▶ When we use parametric equations, the dimension of the ambient space is clear from the size of the vectors. The equation

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + t \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

is a line in \mathbb{R}^2 , while the equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + t \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

is a line in \mathbb{R}^3 .

- ▶ However, the Cartesian equation $x = 0$ is a point if we are in \mathbb{R} , the y -axis if we are in \mathbb{R}^2 and the y - z plane if we are in \mathbb{R}^3 . So you need to know both the equation and the ambient space.

Cartesian equations 2

- ▶ The Cartesian equation for a line in \mathbb{R}^2 is of the form $ax + by + c = 0$.
- ▶ We start with two degrees of freedom in \mathbb{R}^2 , so with one restriction, we get an object of dimension 1 and codimension 1. The codimension of an object is the dimension of the ambient space minus the dimension of the object.
- ▶ However, the corresponding equation in \mathbb{R}^3 , $ax + by + cz + d = 0$, gives us an object in \mathbb{R}^3 of codimension 1 (since we have one equation), namely a plane.
- ▶ So how do we get the Cartesian equation of a line in \mathbb{R}^3 ?
- ▶ A line in \mathbb{R}^3 can be written as the intersection of two planes (in infinitely many ways). So the Cartesian equation of a line in \mathbb{R}^3 will be of the form

$$a_1x + b_1y + c_1z + d_1 = 0$$

$$a_2x + b_2y + c_2z + d_2 = 0.$$

Comparing Parametric and Cartesian equations

- ▶ Notice that parametric equations build objects by adding more spanning vectors, but still just using one equation. If we wanted to describe a 4-dimensional plane in \mathbb{R}^6 , the parametric equation would be of the form

$$\vec{X} = \vec{P} + t_1 \vec{V}_1 + \cdots + t_4 \vec{V}_4.$$

- ▶ However, the Cartesian description would be a system of two equations, since the codimension is 2.

Comparing Parametric and Cartesian equations

- ▶ Explain the painter vs sculptor analogy for the difference between parametric and Cartesian equations.
- ▶ What is codimension?
- ▶ Why is it easier to describe a line in \mathbb{R}^2 than a line in \mathbb{R}^3 using Cartesian equation?
- ▶ Write down the formula for a plane through a point $P = (x_0, y_0, z_0)$ with normal vector $n = (a, b, c)$.
- ▶ How do we compute the angle between a line and a plane?
- ▶ How do we compute the angle between two planes?
- ▶ How do we go between parametric and Cartesian equations for planes?

Comparing Parametric and Cartesian equations

- ▶ What are skew lines (vindskeive linjer)?
- ▶ How do go between parametric and Cartesian equations for lines in \mathbb{R}^2 ?
- ▶ How do go between parametric and Cartesian equations for lines in \mathbb{R}^3 ?