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Solid Geometry

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- A parametric equation of a plane in $\mathbb{R}^n$ is of the form $\vec{X} = \vec{P} + s\vec{U} + t\vec{V}$, where $\vec{X} = \{x_1, \ldots, x_n\}$ is an arbitrary point on the plane, $\vec{P}$ is a given point on the line, and $\vec{U}$ and $\vec{V}$ are spanning vectors of the plane.

Parametric equations 2

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- Notice that we only need one equation to describe either of these objects. However, if we write the equations componentwise, we will get n equations. For example, we can write the equation for a line in $\mathbb{R}^3$ as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + t \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

or as

$$\begin{aligned} x &= p_1 + tv_1 \\ y &= p_2 + tv_2 \\ z &= p_3 + tv_3. \end{aligned}$$

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is a line in $\mathbb{R}^2$, while the equation

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- However, the Cartesian equation $x = 0$ is a point if we are in $\mathbb{R}$, the y-axis if we are in $\mathbb{R}^2$ and the y-z plane if we are in $\mathbb{R}^3$. So you need to know both the equation and the ambient space.

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- So how do we get the Cartesian equation of a line in $\mathbb{R}^3$?
- A line in $\mathbb{R}^3$ can be written as the intersection of two planes (in infinitely many ways). So the Cartesian equation of a line in $\mathbb{R}^3$ will be of the form

$$a_1x + b_1y + c_1z + d_1 = 0$$
$$a_2x + b_2y + c_2z + d_2 = 0.$$

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- However, the Cartesian description would be a system of two equations, since the codimension is 2.

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