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# Laws of Trigonometry

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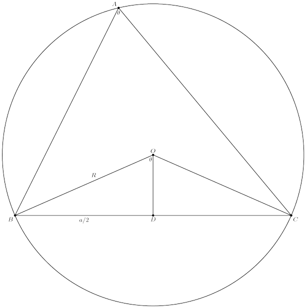
# Extended Law of Sines

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- Let $R$ be the radius of the circumscribed circle. Consider the triangle $\Delta BDO$. Then

$$\sin\theta = \frac{a/2}{R}, \quad \text{so} \quad \frac{a}{\sin\theta} = 2R.$$

# Laws of Trigonometry

- The law of sines may give two possible values if you are trying to determine an angle that could be the largest angle. If you know that the angle is not the largest, you are OK.
- The law of cosines may give two possible values if you are trying to determine a side that is adjacent to the angle. If you use it to determine an angle or the opposite side, you are OK.
- If you have a choice, the law of sines is usually easier to use.

# Solving a Triangle

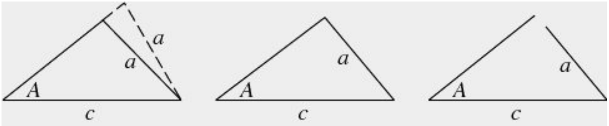
SSS Use the law of cosines for the *largest* angle (the anlge opposite the longest side), use the law of sines for another angle, and then use the sum of the angles rule.

SsA Use the law of sines for the angle opposite the *short* side, use the sum of the angles rule, and then the law of sines for the last side.

SAS Use the law of cosines for the opposite side, the law of sines for the *smaller* angle, and the sum of the angles rule.

ASA Use the sum of the angles rule, and then use the law sines twice.

# sSA

- 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$$a^2 = x^2 + c^2 - 2xc\cos Ax$$

$$x^2 - 2c\cos Ax + c^2 - a^2 = 0$$

$$x = \frac{2c\cos A \pm \sqrt{4c^2\cos^2 A - 4(c^2 - a^2)}}{2}$$

$$x = c\cos A \pm \sqrt{c^2(\cos^2 A - 1) + a^2)}$$

$$x = c\cos A \pm \sqrt{a^2 - c^2\sin^2 A}$$

# sSA 2

- The distance from $B$ to the opposite side is $c\sin A$, which corresponds to $x = b = c\cos A$. If $a < c\sin A$, there will be no solutions, and if $c\sin A < a$, we would expect that there would be two solutions.
- However, if we solve

$$\begin{aligned} c\cos A - \sqrt{a^2 - c^2\sin^2 A} &= 0, \\ c^2\cos^2 A &= a^2 - c^2\sin^2 A, \\ c^2 &= a^2, \end{aligned}$$

we see that if $c\sin A < a < c$, there will be two solutions, but if $c < a$, there will only be one solution, since one intersection is to the left of $A$.