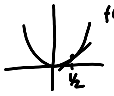
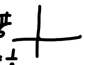
$h(x) = f(g(x)) \quad h'(x) = f'(g(x))\, g'(x)$

$f(x) = x^2 \quad g(x) = \sin x \quad h(x) = (\sin x)^2$

$f(x) = x^2$



$x = \frac{\pi}{6}$



$\sin \frac{\pi}{6} = \frac{1}{2}$

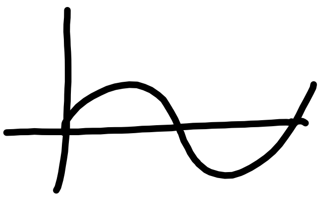
$f'(\frac{1}{2}) = 2(\frac{1}{2}) = 1$

$$h'(x) = \lim_{\Delta x \to 0} \frac{f(g(x+\Delta x)) - f(g(x))}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(g(x+\Delta x)) - f(g(x))}{g(x+\Delta x) - g(x)} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$y = g(x)$

$$= \lim_{\Delta x \to 0} \frac{\Delta f(g)}{\Delta g} \frac{\Delta g}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta f(y)}{\Delta y} \frac{\Delta g}{\Delta x} = f'(y)\, g'(x) = f'(g(x))\, g'(x)$$

vekst i $f(g)$ = (f vekst i g) · (vekst i g)



Kan $g(x) = g(x+\Delta x)$?

$\Delta x = \pi \quad x = 0$

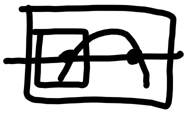
$\sin \pi = \sin 0$

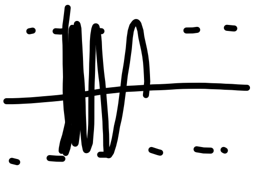
Hvis $\Delta x \approx 0$, så er $\sin(x+\Delta x) \neq \sin x$

Kan vi alltid anta at vi kan si

$$g(x+\Delta x) \neq g(x)$$

bare Δx er liten nok?

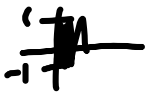


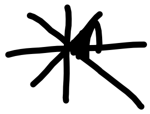


$\sin \frac{1}{x}$ er 0 uendelig mange ganger, vilkårlig nære $x=0$

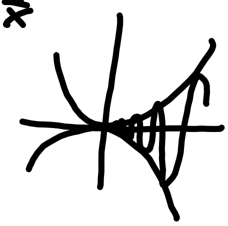
hvis $g(x)$ har isolerte nullpunkter, så OK.

$$f_n(x) = \begin{cases} x^n \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

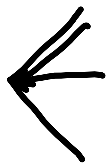
$f_0(x) = \sin \frac{1}{x}$  f_0 ikke kont i 0

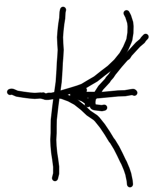
$f_1(x) = x \sin \frac{1}{x}$  $\lim_{x \to 0} f_1(x) = 0$ f_1 er kont i 0.

$f_2(x) = x^2 \sin \frac{1}{x}$



$f_2'(0) = 0$





$f_n(x) = \begin{cases} x^n \sin\frac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$ $\quad \underset{x\neq 0}{f_1'(x)} = \sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x}$

$f_1'(0) = \lim_{x\to 0} \frac{f_1(x)-f_1(0)}{x-0} = \lim_{x\to 0} \frac{x \sin\frac{1}{x}}{x} = \lim_{x\to 0} \sin\frac{1}{x}$ eks. ikke

$f_2'(0) = \lim_{x\to 0} \frac{f_2(x)-f_2(0)}{x} = \lim_{x\to 0} \frac{x^2 \sin\frac{1}{x}}{x} = \lim_{x\to 0} x\sin\frac{1}{x} = 0$

$\underset{x\neq 0}{f_2'(x)} = 2x\sin\frac{1}{x} + x^2\cos\frac{1}{x}\left(-\frac{1}{x^2}\right) =$

$2x\sin\frac{1}{x} - \cos\frac{1}{x}$ $\qquad f_2'(0) \neq \lim_{x\to 0} f_2'(x)$

$\lim_{x\to 0} f_2'(x)$ eks. ikke $\quad f_2'$ er ikke kont. i 0

f_2 er ikke <u>kontinuerlig derverbar</u>

$\frac{\Delta y}{\Delta x}$

f_0 ikke kont

f_1 kont, ikke der

f_2, der, ikke kont der

f_3 ~~2 ganger der, f'~~ f_3' kont, f_3'' eks ikke

$f_3(x) = x^3 \sin\frac{1}{x}$

$f_3'(x) = 3x^2\sin\frac{1}{x} + x^3\cos\frac{1}{x}\left(-\frac{1}{x^2}\right) =$

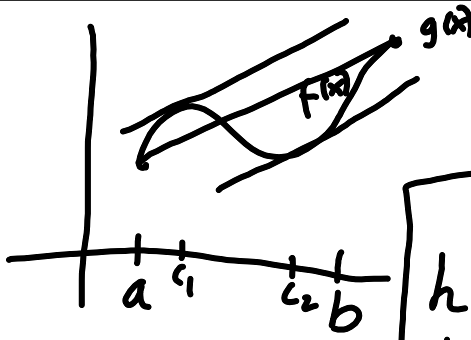
$3x^2\sin\frac{1}{x} - x\cos\frac{1}{x}$

$\lim_{x\to 0} f_3'(x) = 0 = f_3'(0) \quad f_3'$ kont

$f_3''(0) = \lim_{x\to 0} \frac{f_3'(x) - f_3'(0)}{x} = \lim_{x\to 0}\left(3x\sin\frac{1}{x} - \cos\frac{1}{x}\right)$

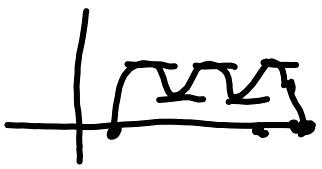
eks ikke!

f_4 f_4'' eks, f_4'' er ikke kont.



$$\frac{f(b)-f(a)}{b-a} = \frac{\Delta y}{\Delta x}$$

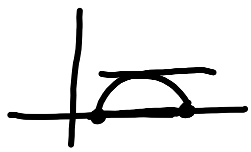
$$f(a) = f(b) = 0$$

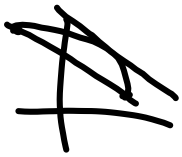


$$g = \frac{f(b)-f(a)}{b-a}(x-a) + f(a)$$

$$h(x) = f(x) - g(x)$$

$$h(a) = h(b) = 0$$





$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$ voksende

$\Rightarrow f(x_1) < f(x_2)$ strengt voksende

$$f(x) \geq 0 \Rightarrow \lim_{x \to a} f(x) \geq 0$$

$$f(x) > 0 \not\Rightarrow \lim_{x \to a} f(x) > 0$$

$$a_n = \frac{1}{n} > 0 \quad \lim_{n \to \infty} \frac{1}{n} = 0$$

f er voksende på (a,b) (eller strengt voks.)

$c \in (a,b)$

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \geq 0$$

anta at $h > 0$

$h < 0$

$f(x) = x^3$ $c = 0$ $f'(0) = \lim_{h \to 0} \frac{h^3 - 0}{h} = \lim_{h \to 0} h^2 = 0$

anta at $f' > 0$ på (a,b).

Velg $x_1 < x_2$ $x_1, x_2 \in [a,b]$

Se på $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(c) > 0$

for en $c \in (x_1, x_2)$ $\Downarrow$

$f(x_1) < f(x_2)$

$\lim_{x\to\infty} \sin\frac{1}{x} = 0$ $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to\infty} x \sin\frac{1}{x} = \lim_{y\to 0} \frac{1}{y} \sin y = 1$

$y=\frac{1}{x}$ $x=\frac{1}{y}$

$x\to\infty \leftrightarrow y\to 0$

Du beveger deg langs $y=x^3$ med konstant fart.
Tangentvektoren måler din hastighet

solvere

hvarf – vende, snu

solstice, stand – stå

至 zhì ~~ekst~~ ekstrem

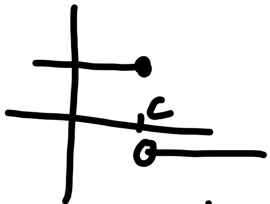
Er $f'(a) = \lim_{x\to a} f'(x)$? NOT!

Dette er def. av kont. deriverbar $\Leftrightarrow$
f' er kont.

$x^2 + x^2 \sin\frac{1}{x} = x^2\left(1+\sin\frac{1}{x}\right)$

$2x^2 + x^2 \sin\frac{1}{x} = x^2\left(2+\sin\frac{1}{x}\right)$

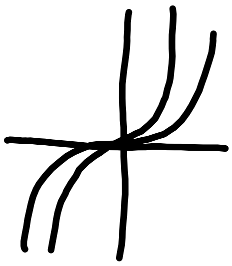
Hvis $f'(c)>0$ og f' er kont, da er det

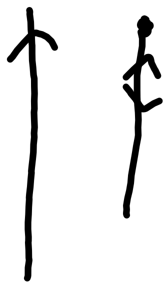


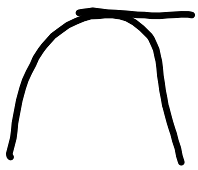
$f(c)>0$, f er ikke pos. på en interval rundt c.

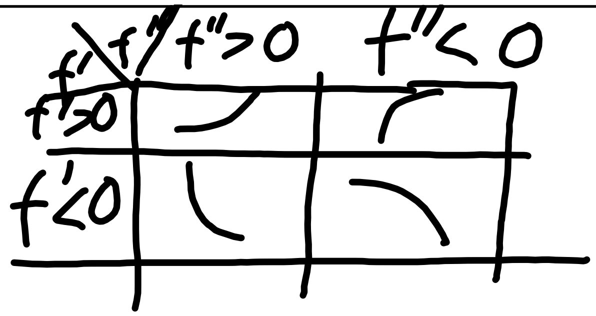
f er kont og $f(c)>0 \Rightarrow f>0$ på et interval rundt c

$f(c)=\lim_{x\to c} f(x)$



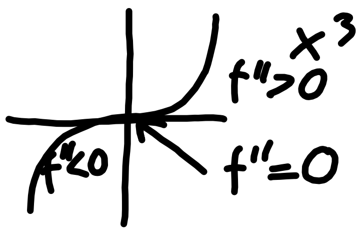






$f''>0$ krummer opp

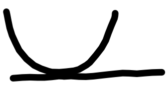
$f''<0$ krummer ned



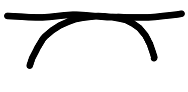
x^3+x

c er et vendepunkt $\not\Rightarrow f'(c)=0$

$f'(c)=0$ & $f''(c)>0$

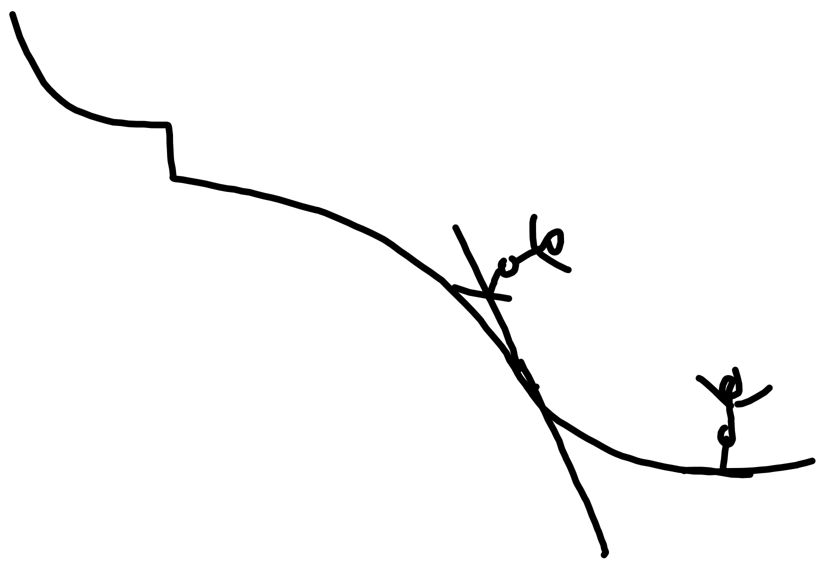


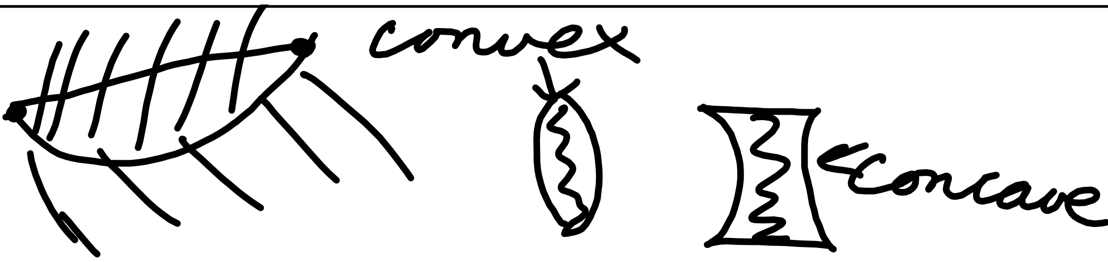
$f'(c)=0 \wedge f''(c)<0$



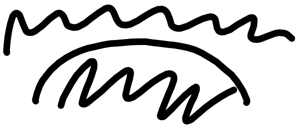
$f'=f''=0 \not\Rightarrow$ vendepunkt $f(x)=x^4$

$f(x) = x^4$
$f'(x) = 4x^3$
$f''(x) = 12x^2 \geq 0$
$f'(0) = f''(0) = 0$





convex down = concave up



concave ~~up~~ down = convex ~~down~~ up

pl. of infl = change of concavity

ekstrem punkt = monotonitet skifte

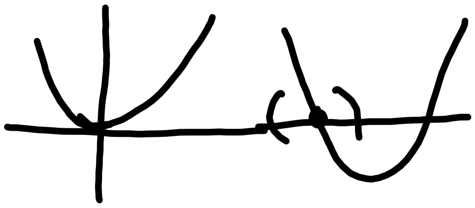
$\not\Rightarrow$ $2x^2+x^2\sin\frac{1}{x}$

vendepunkt = concavitet skifte

f'' skifter fortegn

f' skifter fortegn

moteksempler: x^4 x^3

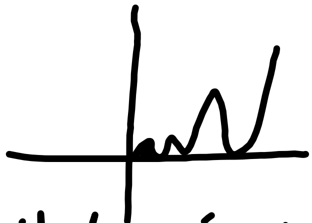


kysser   krysser

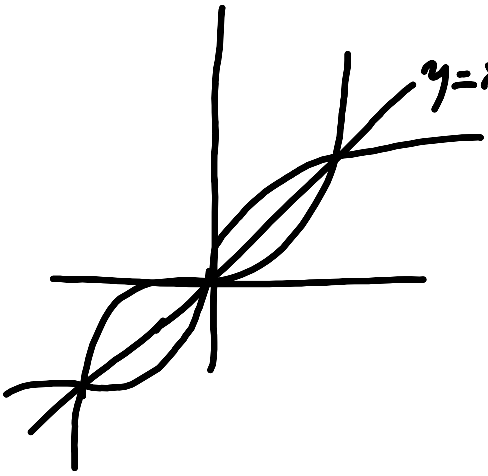
kissing problem

→ skifter ikke fortegn

$x^2 + x^2 \sin \frac{1}{x}$



ikke-isolert nullpunkt



x^3 $\sqrt[3]{x} = x^{1/3}$

$y = ax + b$

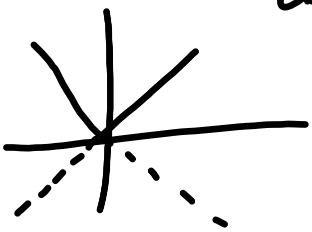
$ax + by + c = 0$

<u>alle</u> linjer i planet

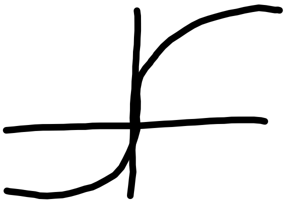
Hvis $b \neq 0$ $y = -\frac{a}{b}x - \frac{c}{b}$

$b = 0$ $ax + c = 0$ $x = -\frac{c}{a}$

$f(x)=|x|$ ikke deriverbar fordi
ikke tangent



$f(x)=x^{1/3}$

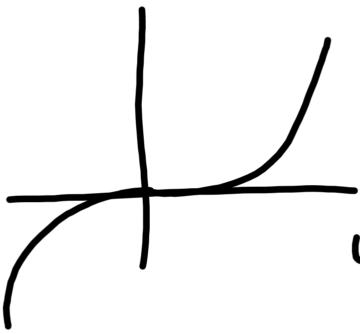


har tangent, men tangenten
har ikke stigningstall
<u>ikke</u> deriverbar
vendepunkt, men
$f'(0)$ og $f''(0)$ eks. <u>ikke</u>

$$f(x)=\begin{cases} x^2 & x\geq 0 \\ -x^2 & x<0 \end{cases} \qquad f'(x)=\begin{cases} 2x & x\geq 0 \\ -2x & x<0 \end{cases}$$

$$f''(x)=\begin{cases} 2 & x>0 \\ -2 & x<0 \end{cases}$$

$f''(0)$ eks ikke



vendepunkt

Grenseverdier bevarer ikke strenge ulikheter. $a_n = \frac{1}{n} > 0$ $\lim_{n\to\infty} \frac{1}{n} = 0$

$f(x) = x^3$ $f'(x) = 3x^2$

1) f strengt voksende, men $f' \not> 0$

2) $f'(0)=0$, men 0 er ikke et ekstremp. (terrassep.)

$f(x) = x^4$

$f''(0)=0$ men ikke vendep.

vendep i c $\wedge$ $f''(c)$ eks $\Rightarrow f''(c)=0$

anta at $f''(x) > 0$ for $x > c$

og $f''(x) < 0$ for $x < c$

$$f''(c) = \lim_{x\to c} \frac{f'(x)-f'(c)}{x-c}$$

1) $x > c$. $\exists d, c<d<x$ slik at $\frac{f'(x)-f'(c)}{x-c} = f''(d) > 0$

$$\lim_{x\to c^+} \frac{f'(x)-f'(c)}{x-c} \geq 0$$

2) $x<c$ $\exists d$, $x<d<c$, s.a. $\frac{f'(x)-f'(c)}{x-c} = f''(d) < 0$

$$\lim_{x\to c^-} \frac{f'(x)-f'(c)}{x-c} \leq 0$$

$\Rightarrow f''(c)=0$ siden $f''(c)$ eks.

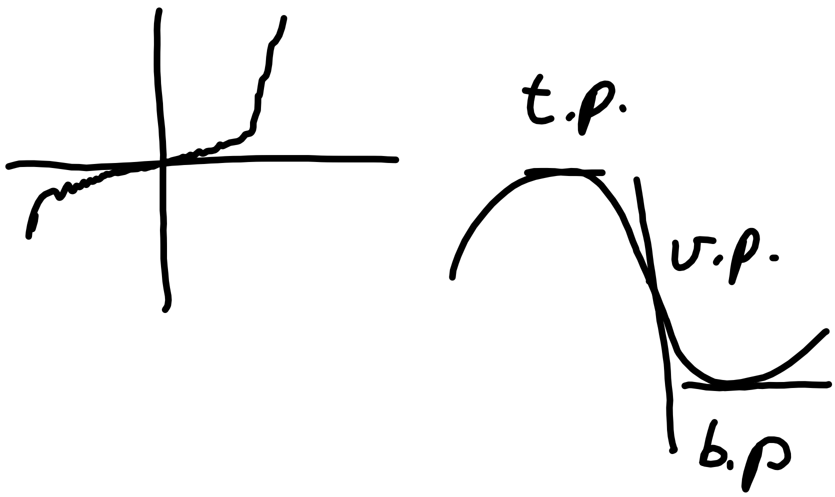
f har v. punkt → du vet noe om f'' rundt c.

↓

du vet noe om f' rundt c via MVT (MVS)

↓

du vet noe om f'' i c (def. av der)



$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1$$

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \lim_{x\to 0} \frac{\frac{f(x)-f(0)}{x-0}}{\frac{g(x)-g(0)}{x-0}} = \lim_{x\to 0} \cancel{\frac{f'(c)}{g'(c)}} = L$$

~~Cauchys MVT~~ ~~c mellom x og 0.~~

Anta at f og g er kont. deriv.

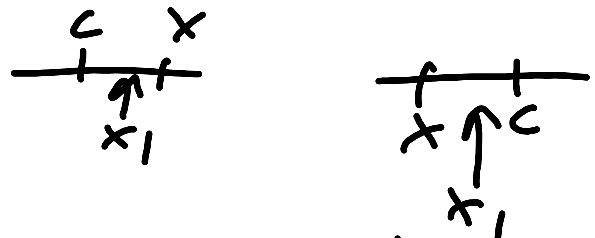
$$\frac{\lim_{x\to 0} \frac{f(x)-f(0)}{x-0}}{\lim_{x\to 0} \frac{g(x)-g(0)}{x-0}} = \frac{f'(0)}{g'(0)} = \lim_{x\to 0} \frac{f'(x)}{g'(x)}$$

siden f og g er kont. d.

Tangentlinjen

$f(c)+f'(c)(x-c) \approx f(x)$ for x nær c

$f(c)+f'(x_1)(x-c) = f(x)$



$x-c$ og x_1-c har samme fortegn!

$x^n + x^k \sin\left(\frac{1}{x}\right)$

f'' skifta ikke fortegn fordi den skifter fortegn hele tiden!

G,G
G,J
J,J

$P(G,J) = \frac{1}{3}$, $P(G,G) = P(J,J) = \frac{1}{\cancel{4}\,3}$. NOT

$\parallel$ 1/2 $\qquad \parallel \frac{1}{4}$

$P(G,J) = \frac{1}{2}$

	G	J
G		
J		

x	xx
	x

$\{0,1\} = \{1,0\} = \cancel{\{0,0,1,1\}}$

$\binom{7}{5} = \binom{7}{2}$

uordnet ↔ like

med tilbakel. ↔ gjentagelse

$\binom{n}{k}$ 3 baller 5 farger

ulik farge

$\binom{5}{3} = \frac{5\cdot4\cdot3}{6} = 10$ $\binom{5+3-1}{3} = \binom{7}{3} = \frac{7\cdot6\cdot5}{6} = 35$

Kombinatorikk = antall mulige utvalg
↑
helt tall

Sannsynlighet = tall mellom 0 og 1

1. og 2. premie i matematikk → uten tilb ordnet

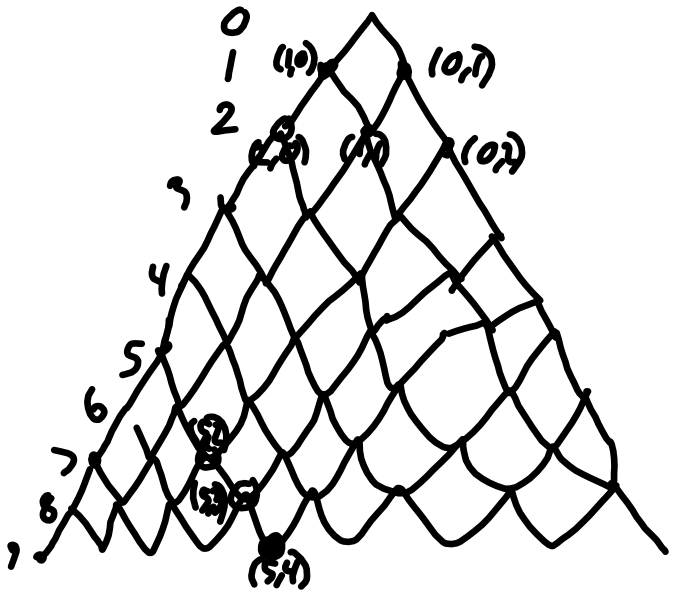
To like premier til de to beste i matematikk → uten tilb uordnet

Premie for flinkest i engelsk &
– " – matematikk med tilbak.

Ulike premier ↔ ordnet Like premier ↔ uord.

(A)	1/2	(A, A)	1/4
(B, A)	1/4	(A, B)	1/4
(B, B)	1/4	(B, A)	1/4
		(B, B)	1/4

antall omganger



$r=1\ ,\ S=2$

A: $\frac{1}{4}\left(\binom{2}{0}+\binom{2}{1}\right)=\frac{1}{4}(1+2)=\frac{3}{4}$

B: $\frac{1}{4}\binom{2}{2}=\frac{1}{4}$

$r=2\ ,\ S=5$

A har ~~vel~~ vunnet 4
B — " — 1

(4,1)

1 (A,A,A,A,A,A)
6 (A,A,A,A,A,B), (A,A,A,A,B,A).....
$\binom{6}{2}=15$

6 (A,B,B,B,B,B), (B,A,B,B,B,B), ...
1 (B,B,B,B,B,B)

A: $\frac{2^6-7}{2^6}$

B: $\frac{7}{2^6}$

200 familier

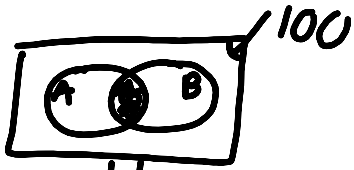
50 (G,G) ~~50~~ (G,J) ~~50~~ (J,G) ~~50 (J,J)~~
25 25

$P(G,G) = \frac{1}{2}$

P({G,J} | Minst én gutt)

q = P(Minst én gutt | {G,J})

Kjenner $P(A|B)$, vil finne $P(B|A)$



$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

$P(B|A) = P(B)$

A&B uavh

$$P(A\cap B) = P(B|A)P(A)$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$\rightarrow P(A|B)P(B)$

$|B| = 20$

$|A| = 30$

$|A\cap B| = 10$

$P(B|A) = \frac{10}{30} = \frac{1}{3}$

$P(B) = \frac{20}{100} = 0{,}2 \neq \frac{1}{3}$

$$\boxed{P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{P(A|B)P(B)}{P(A)}}$$

$X \cup Y =$ utfallsrommet

$X \cap Y = \emptyset$

$$P(A) = P(A\cap X) + P(A\cap Y) = P(A|X)P(X) + P(A|Y)P(Y)$$

Får barn til man har fått en sønn.
Stopper etter 4.

G	$1/2$	$= 8/16$
J, G	$1/4$	$= 4/16$
J, J, G	$1/8$	$= 2/16$
J, J, J, G	$1/16$	
J, J, J, J	$1/16$	
	1	

$$G: 1\cdot\frac{1}{2} + 1\cdot\frac{1}{4} + 1\cdot\frac{1}{8} + 1\cdot\frac{1}{16} = \frac{15}{16}$$

$$J: 0\cdot\frac{1}{2} + 1\cdot\frac{1}{4} + 2\cdot\frac{1}{8} + 3\cdot\frac{1}{16} + 4\cdot\frac{1}{16} = \frac{4+4+3+4}{16} = \frac{15}{16}$$

annen derivert
andrederivert

annengradslligning
andregradslligning

han / ham
he / him

annen ~~andre~~ andre
second other

Uniform?
Valg?

1) 3 kort

~~S/S~~ H/H S/H

S/H | S/H = S/H

Du trekker H, Er det H på den andre siden også?

a) H/H og S/H like sanns. ⇒ $P = 1/2$

b) 2/3 for at kortet har to like sider ⇒ $P = 2/3$

b2) 1: S_1/S_2 2: H_1/H_2 3: S/H

~~$(1,S_1)$~~, ~~$(1,S_2)$~~, $(2,H_1)$, $(2,H_2)$, ~~$(3,S)$~~, $(3,H)$

---

2) Principle of restricted choice

Øst har kastet en konge. Har Øst K,D eller bare K?

Ø = K

Ø = K,D og spiller K
Ø = K,D og spiller D
= Ø = K,D

$P(Ø=K,D \mid Ø \text{ spilte } K) = 1/3$

$P(Ø=K,D) = 1/2$

K	☒

← K,D + K
← K,D + D

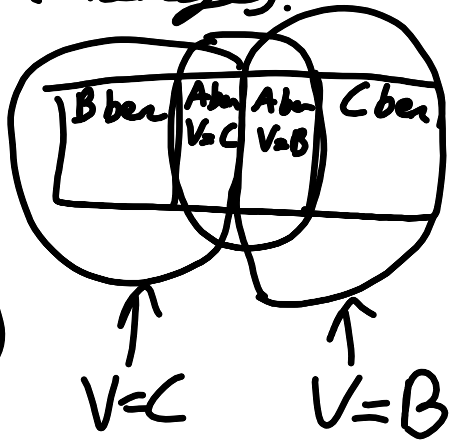
3) 3 fanger

Vokter forteller A at B skal henges.

A ben. C ben.

P = 1/2

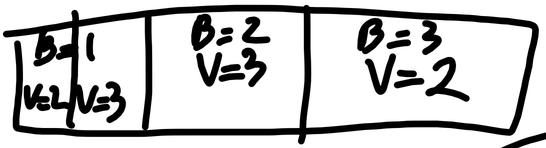
$$\frac{(A \text{ ben} \mid \text{Vokter sier } B)}{(A \text{ ben} \mid \text{Vokter sier } C)}$$

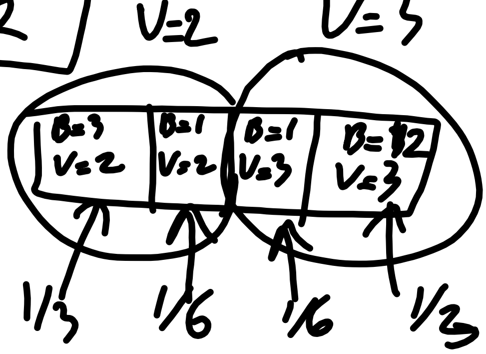


P(A ben | Vokter sier B) = $\frac{1}{3}$

4) Monty Hall

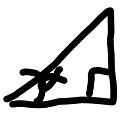
3 fanger — 3 dører
guvernør — deltaker
vokter — vert



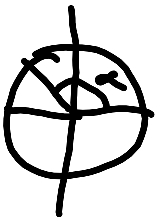


Bilen bak dør i.
B=i
Spiller velger dør 1

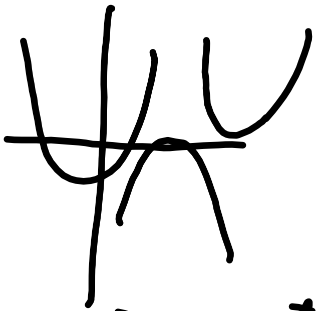
jiva
jiba jb
jaib - bukt
komplementær vinkel 90−x
$\cos x = \sin(90-x)$
cosinus er sinus til komplementær v.

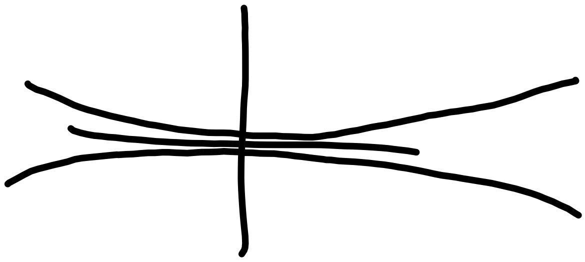


$0 < \alpha < 90$



$0 \leq \alpha \leq 180$





$p(x)$ er et polynom

$p(a) = 0 \iff (x-a) \mid p(x)$

$p(x) = (x-a)\,q(x)$

$p(a) = (a-a)\,q(a) = 0$

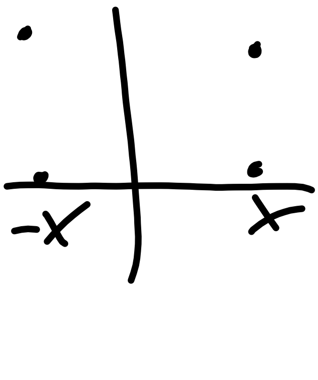
$p(x) = a(x-x_1)(x-x_2)$

f er sym om y-aksen

$\Updownarrow$

$f(-x) = f(x)$

$f(x) = ax^2 + bx + c$



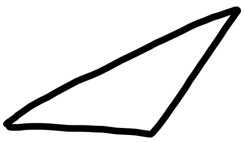
a $\quad ax^2 + bx + c$

$a(x-x_1)(x-x_2) =$

$ax^2 - a(x_1+x_2)x + ax_1x_2$

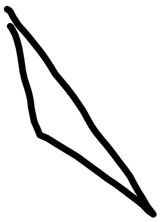
a og b har forskj. fortegn $\Leftrightarrow$

$x_1 + x_2 > 0$



3 sider

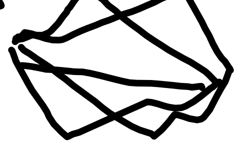
3 vinkler

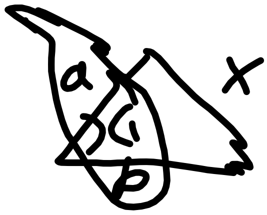


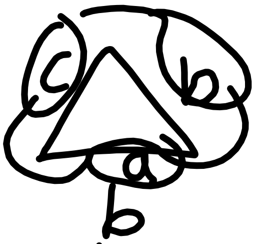
rettvinklet Δ

hyp

høyden 3

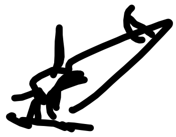






$$x^2 = a^2 + b^2 - 2a\cancel{b}\cos C$$

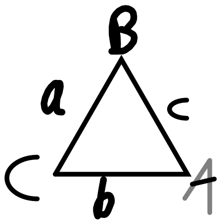
$$c^2 = x^2 + b^2 - 2b\cos C\, x$$



vinkelen mellom horisonten og solens bane er 90 − breddegrad

The tropics is the area between the Tropics.

Steradianer − vinkelmål i R^3



sin 2 sider og 2 vinkler

sider a, b

vinkel C

cos 3 sider og 1 vinkel

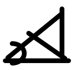
a, b, C 2 sider og mellomliggende v.

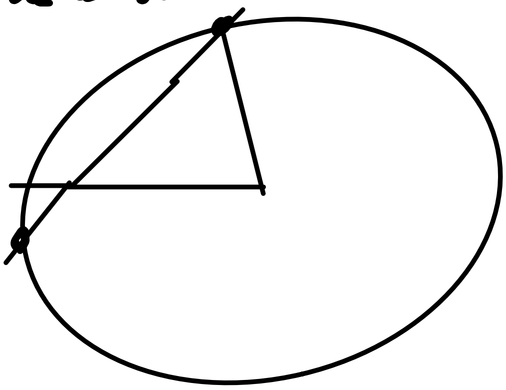
a, c, C 2 sider og motstående v. til en av sidene

SSS

SAS 2 sider og mellomliggende vinkel

SsA 2 sider og motstående vinkel til den lengste av de to sidene

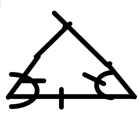




SsA OK

sSA to mulige

ASA



$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin A}{\sin a} = \frac{\sin C}{c}$

Gjennomsnitt er ikke lett!

1998 Før siste runde

Jordan 2313 poeng 81 kamper

$2313/81 \approx 28.6$

Shaq $1666/59 \approx 28.2$

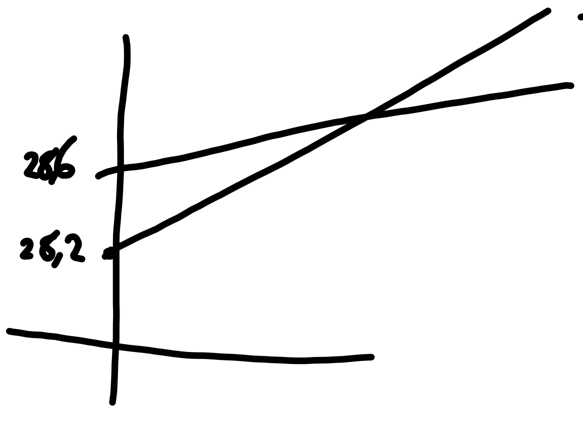
anta at Jordan skorer j i siste kamp

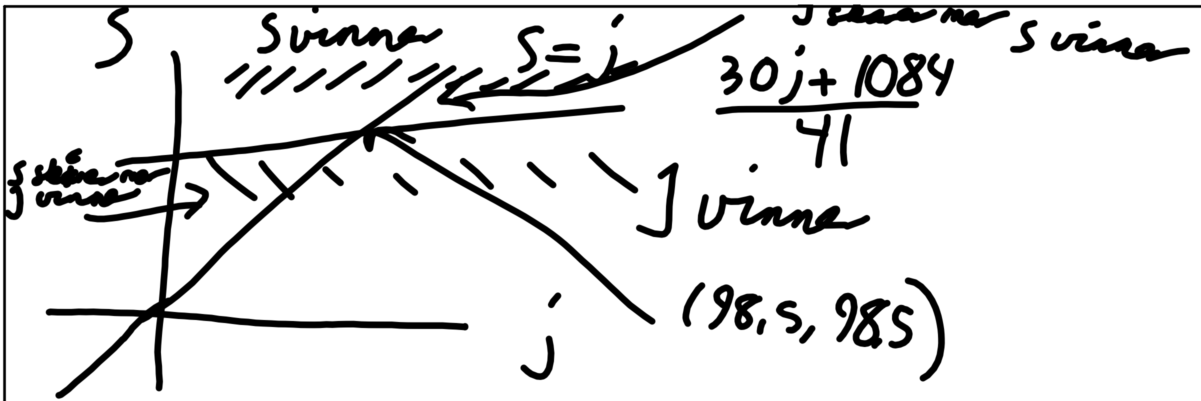
Shaq. s

J vinner hvis $\frac{2313+j}{82} > \frac{1666+s}{60}$

$\frac{s}{60} + \frac{1666}{60} \approx \frac{s}{60} + 28.2$

$\frac{2313+j}{82} = \frac{j}{82} + \frac{2313}{82} \approx \frac{j}{82} + 28.6$





$$\frac{2313+j}{82} = \frac{1666+s}{60}$$

$$s = \frac{30j+1084}{41}$$

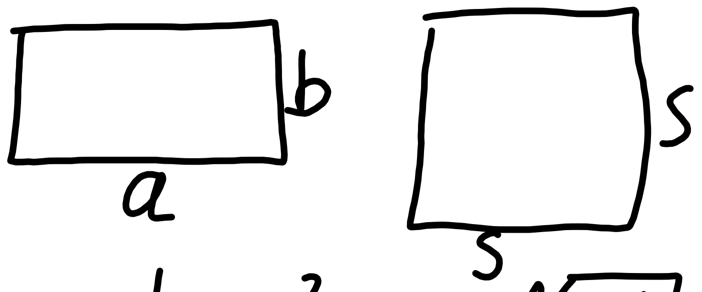
Til hytta 30 km/t
Fra hytta 60 km/t
avstand d km
ut v km/t
hjem w km/t

$s = s \cdot t \cdot us$

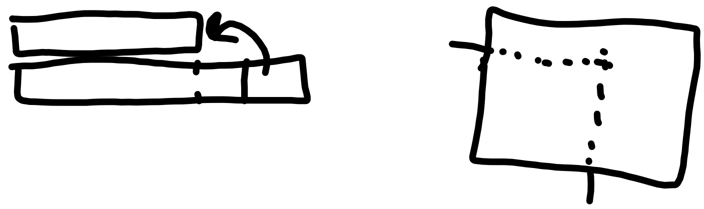
$v = s/t \quad t = \frac{s}{v}$

$$v = \frac{s}{t} = \frac{2d}{\frac{d}{v} + \frac{d}{w}} = \frac{2}{\frac{1}{v} + \frac{1}{w}} = H(v,w)$$

Harmoniske gjennomsnitt av v og w.
Aritmetiske – " – $A = \frac{v+w}{2}$
Geometriske – " – $G = \sqrt{v \cdot w}$



$ab = s^2 \quad s = \sqrt{ab}$



$\frac{a+b}{2} \qquad \sqrt{ab}$

$0 \cdot a = 0 \qquad a^0 = 1$

Den tomme sum — Det tomme produkt

$na \longleftrightarrow a^n$

$a + \cdots + a \qquad a \cdot \cdots \cdot a$

$\frac{1}{2}(a+b) \longleftrightarrow (ab)^{1/2}$

6 $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+ \dots$

$$a_n = \frac{2}{\frac{1}{a_{n-1}}+\frac{1}{a_{n+1}}} = \cancel{\frac{2}{\frac{1}{n-1}+\frac{1}{n+1}}} = \cancel{\frac{2(n-1)(n+1)}{n+1+n-1}} =$$

$a_n = \frac{1}{n}$

$$\cancel{\frac{2(n^2-1)}{2n}}$$

$$\frac{2}{\frac{1}{\frac{1}{n-1}}+\frac{1}{\frac{1}{n+1}}} = \frac{2}{n-1+n+1} = \frac{2}{2n} = \frac{1}{n}$$

$a, a+d, a+2d, a+3d, \dots$

$$a_n = \frac{1}{2}(a_{n-1}+a_{n+1})$$

$a, ar, ar^2, ar^3, \dots$

$$a_n = \sqrt{a_{n-1}a_{n+1}} = \sqrt{\left(\frac{1}{r}a_n\right)(r a_n)} = \sqrt{a_n^2} = a_n$$

Følge a_n → Rekke $\sum a_n$

$a_n = \frac{1}{n^2}$ $\qquad \sum a_n = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$

Følgen av delsummer ← Rekke $\sum a_n$

$$S_n = \sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{\infty} 9\left(\frac{1}{10}\right)^k = 0{,}9 + 0{,}09 + \dots$$

$S_1 = 0{,}9$

$S_2 = 0{,}99$

$S_3 = 0{,}999$

$\vdots$

$S_n \to 1$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

$$1 + x + x^2 + \dots = \frac{1}{1-x}$$

$$S_n = \sum_{k=1}^{n} a_n \qquad S_n = \sum_{k=1}^{n} a_k$$

$$F(x) = \int_a^x f(t)\,dt$$

$$F(x) = \int_a^x f(x)\,dx$$

$x = b$

$$F(b) = \int_a^b f(b)\,db$$

$f(x) = x^2 + 1 \qquad x^2 + 1 = 0$

$f(x) = 0$

$f(x) \equiv 0$

$f(x) := 0$

$x = x+1$ $\qquad x_{n+1} = x_n + 1$

$x := x+1$

$\sum a_n \quad S_n = \sum_{k=1}^{n} a_k$

$S_1 = a_1 \quad S_2 = a_1 + a_2 \quad S_3 = a_1 + a_2 + a_3$

$S_3 = S_2 + a_3$

$a_n = S_n - S_{n-1} \longleftarrow S_n = S_{n-1} + a_n$

Start med $\{S_n\}_{n \geq 1}$

Set $S_0 = 0$

$\{S_n\}_{n \geq 0}$ $\qquad a_n = S_n - S_{n-1}$

$\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n =$

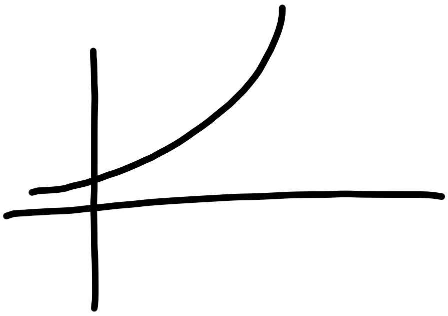
$(S_1 - S_0) + (\cancel{S_2} - \cancel{S_1}) + (S_3 - \cancel{S_2}) + \cdots + (S_n - S_{n-1}) =$

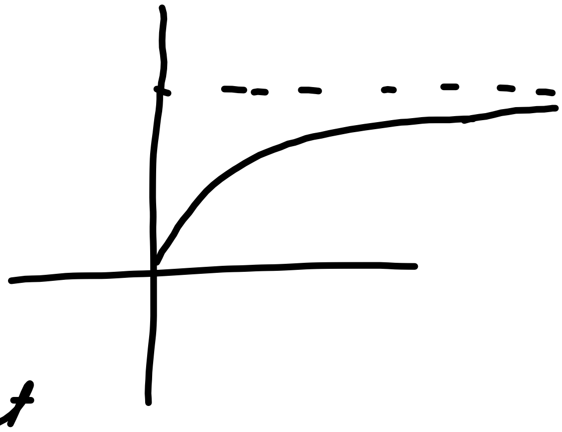
$S_n - S_0 = S_n$

Start med en følge $\{S_n\}_{n \geq 1}$. Set $S_0 = 0$ og se på følgen $\{S_n\}_{n \geq 0}$. Set $a_n = S_n - S_{n-1}$ (Jeg trenger $S_0 = 0$ for å definere a_1)

Da er følgen av delsummer til rekken $\sum_{k=1}^{\infty} a_k$ lik følgen S_n.

Zenos paradokse





t_0= start tidspunkt

t_1= A er der Skilpadden startet

t_2= A er der — " — var ved tid t_1

⋮

t_n= A — " — t_{n-1}

t_n er voksende. Hvis $t_n \to \infty$ så vinner Padden

Padden er 10 m foran

Jeg løper 10 m/s

Padden 1 m/s

$t_0 = 0s \quad t_1 = 1s \quad t_2 = 1{,}1s \quad t_3 = 1{,}11s$

$t_n \to \frac{10}{9}$

$A(t) = 10t \quad S(t) = 1t$

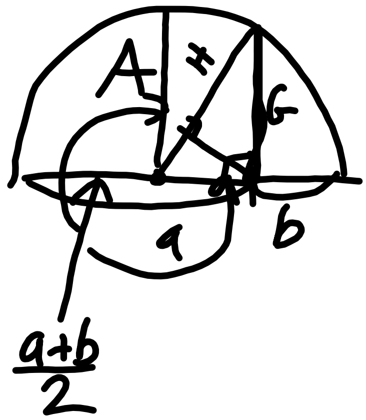
$A(t) = S(t) + 10$

$10t = 1t + 10$

$10t - 1t = 10$

$9t = 10 \quad t = \frac{10}{9}$

$A \geq G \geq H$



$$\left(\frac{a+b}{2}\right)^2 = G^2 + \left(a - \frac{a+b}{2}\right)^2$$

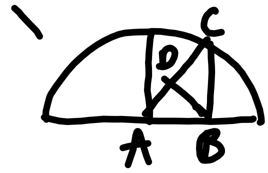
$$\frac{(a+b)^2}{4} = G^2 + \left(\frac{a-b}{2}\right)^2$$

$$(a+b)^2 = 4G^2 + (a-b)^2$$

$$a^2+2ab+b^2 = 4G^2 + a^2-2ab+b^2$$

$$4ab = 4G^2$$

$$G = \sqrt{ab}$$



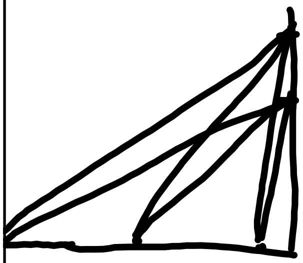
$\triangle ABC \sim \triangle BCD$

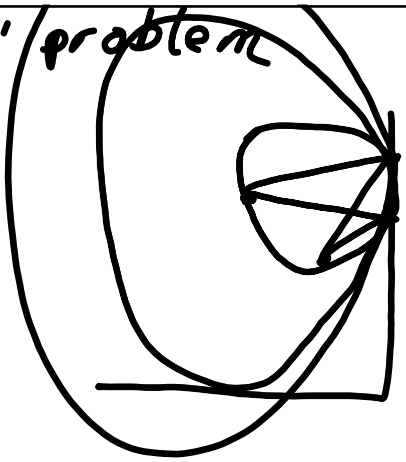
hyp $\dfrac{A}{G} = \dfrac{G}{H}$ ← kat i stor
hyp ← kat i liten

$$AH = G^2$$

$$H = \frac{G^2}{A} = \frac{(\sqrt{ab})^2}{\frac{a+b}{2}} = \frac{ab}{\frac{a+b}{2}} = \frac{2}{\frac{a}{ab}+\frac{b}{ab}} = \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

Regiomontanus' problem



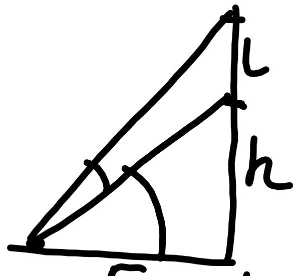


vinkel konstant på sirkel

større sirkel

$\Downarrow$

mindre vinkel



$$f(r) = \arctan \frac{l+h}{r} - \arctan \frac{h}{r}$$

$$f'(r) = \frac{1}{1+\left(\frac{l+h}{r}\right)^2}\left(\frac{-1(l+h)}{r^2}\right) - \frac{1}{1+\left(\frac{h}{r}\right)^2}\left(\frac{-1\cdot h}{r^2}\right) =$$

$$\frac{h}{r^2+h^2} - \frac{l+h}{r^2+(l+h)^2} = 0$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$h(r^2+(l+h)^2) = (l+h)(r^2+h^2)$$

$$hr^2 + h(l+h)^2 = (l+h)r^2 + (l+h)h^2$$

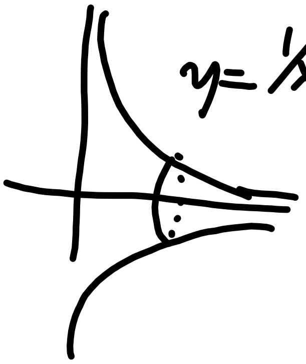
$$h(l+h)^2 - (l+h)h^2 = (l+h)r^2 - hr^2$$

$$lr^2 = h(l+h)(l+h-h) = h(l+h)l$$

$$r = \sqrt{h(l+h)}$$

Gabriels horn

1641 Torricelli



$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

$y = f(x)$

$$\text{Volum} = \pi \int_a^b f(x)^2 \, dx$$

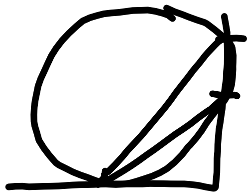
$$\text{areal} = \int_a^b f(x)\sqrt{1+(f'(x))^2} \, dx$$

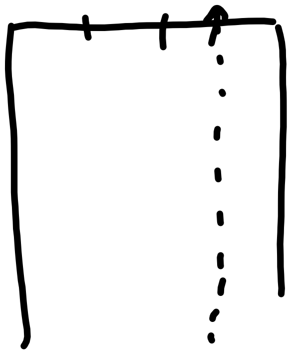
$$\int_1^{\infty} \frac{dx}{x^n} \qquad n>1 \text{ konv.} \qquad 0<n\le 1 \text{ div.}$$

$$f(x) = \frac{1}{x} \qquad V = \pi \int_1^{\infty} \frac{dx}{x^2} \text{ konv.}$$

$$A = \int_1^{\infty} \frac{1}{x} \sqrt{\;} \, dx > \int_1^{\infty} \frac{dx}{x} \text{ div}$$

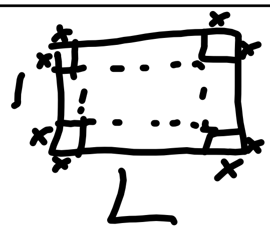
÷10

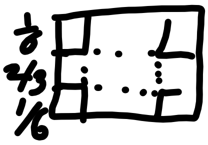




arealformler – potens 2 $4\pi r^2$

volumformler – potens 3 $\frac{4}{3}\pi r^3$





areal av bunnen:

$\frac{2}{3}\cdot\frac{2}{3}=\frac{4}{9}$

areal av sideveggen:

$\frac{1}{6}\cdot 4\cdot\frac{2}{3}=\frac{4}{9}$

$x(1-2x)(L-2x)$

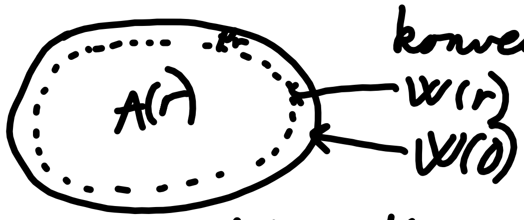
$= V(x,L)$

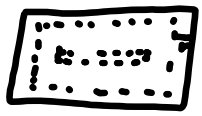
$V'(x,L)=0$

$x=\frac{L+1-\sqrt{(L+1)^2-3L}}{6}$

$L=1 \quad x=\frac{1+1-\sqrt{2^2-3}}{6}=\frac{2-1}{6}=\frac{1}{6}$

konveks





$A(r) =$ arealet av figuren begrenset av kurven $W(r)$

$P(r) =$ omkrets av kurven $W(r)$

$A'(r) = -P(r)$

$V(r) = A(r) \cdot r$

$V'(r) = A'(r) \cdot r + A(r) = A(r) - P(r) \cdot r = 0$

areal av bunn

areal av sidevegg