

Poker Hands

Straight flush Each straight flush is uniquely determined by its highest-ranking card. These ranks go from 5 (A-2-3-4-5) up to A (10-J-Q-K-A) in each of the four suits.

$$\binom{10}{1} \binom{4}{1} = 40$$

Four of a kind Any one of the thirteen ranks can form the four of a kind by selecting all four of the suits in that rank. The final card can have any one of the twelve remaining ranks, and any suit.

$$\binom{13}{1} \binom{4}{4} \binom{12}{1} \binom{4}{1} = 624$$

Full house The full house comprises a triple (three of a kind) and a pair. The triple can be any one of the thirteen ranks, and consists of three of the four suits. The pair can be any one of the remaining twelve ranks, and consists of two of the four suits.

$$\binom{13}{1} \binom{4}{3} \binom{12}{1} \binom{4}{2} = 3,744$$

Flush The flush contains any five of the thirteen ranks, all of which belong to one of the four suits, minus the 40 straight flushes.

$$\binom{13}{5} \binom{4}{1} - 40 = 5,108$$

Straight The straight consists of any one of the ten possible sequences of five consecutive cards, from 5-4-3-2-A to A-K-Q-J-10. Each of these five cards can have any one of the four suits. Finally, as with the flush, the 40 straight flushes must be excluded.

$$\binom{10}{1} \binom{4}{1}^5 - 40 = 10,200$$

Three of a kind Any of the thirteen ranks can form the three of a kind, which can contain any three of the four suits. The remaining two cards can have any two of the remaining twelve ranks, and each can have any of the four suits.

$$\binom{13}{1} \binom{4}{3} \binom{12}{2} \binom{4}{1}^2 = 54,912$$

Two pair The pairs can have any two of the thirteen ranks, and each pair can have two of the four suits. The final card can have any one of the eleven remaining ranks, and any suit.

$$\binom{13}{2} \binom{4}{2}^2 \binom{11}{1} \binom{4}{1} = 123,552$$

Pair The pair can have any one of the thirteen ranks, and any two of the four suits. The remaining three cards can have any three of the remaining twelve ranks, and each can have any of the four suits.

$$\binom{13}{1} \binom{4}{2} \binom{12}{3} \binom{4}{1}^3 = 1,098,240$$

No pair A no-pair hand contains five of the thirteen ranks, discounting the ten possible straights, and each card can have any of the four suits, discounting the four possible flushes. Alternatively, a no-pair hand is any hand that does not fall into one of the above categories; that is, the complement of the union of all the above hands, where the universe is any way to choose five out of 52 cards.

$$\left[\binom{13}{5} - 10 \right] \left[\binom{4}{1}^5 - 4 \right] = 1,302,540$$