Assignment 2 for MAT4170
Spline methods, Spring 2021

To be completed by Tuesday 9 March. Please send your solution as a single pdf file (including plots/figures) to `michaelf@math.uio.no`.

1. State the smoothness properties of $B[0,0,0,0,1](x)$ and $B[0,1,1,1,2](x)$.

2. Use induction on the degree d to show that the formula

$$B[t_j,\dots,t_{j+d+1}](t_i) = B[t_j,\dots,t_{i-1},t_{i+1},\dots,t_{j+d+1}](t_i)$$

   holds for $i = j,\dots,j+d+1$. Use it to prove property (5) of Lemma 2.3.

3. Do problem 2.17.

4. Do problem 2.18. Test your implementation on the knot vector

$$\mathbf{t} = (-1,-1,-1,-1,0,1,2,3,4,5,5,5,5) \tag{1}$$

   by plotting the non-zero B-splines of degree $d = 3$ on the interval $[0,1]$. Hint: do **not** use the B-spline matrices $R_k(x)$. Start with a vector B of zeros of length $d+1$ and set its last element to $B_{\mu,0}(x) = 1$. Then use the recurrence relation (2.1) to compute $B_{j,k}(x)$ for $j = \mu-k+1,\dots,\mu$ for each $k = 1,\dots,d$ (as in Figure 2.8), storing the result in B.

5. Do problem 2.19. Test your implementation by plotting the cubic spline with knot vector $\mathbf{t}$ of (1) and coefficients

$$\mathbf{c} = (-1,1,-1,1,-1,1,-1,1,-1).$$

   Hint: do **not** use the B-spline matrices $R_k(x)$. Start with the vector $\mathbf{c} = (c_{\mu-d},\dots,c_\mu)$. Then compute c_{j-k+1} by (2.20) for $j = \mu,\dots,\mu-k+1$ for each $k = 1,\dots,d$ (as in Figure 2.9), storing the result in $\mathbf{c}$.

6. Do problem 2.26.