Assignment 3 for MAT4170
Spline methods, Spring 2021

To be completed by Tuesday 6 April. Please send your solution as a single pdf file (including plots/figures) to `michaelf@math.uio.no`.

1. Problem 3.2.

2. Problem 4.5. Hint: Insert $d+1$ knots at $a < t_1$ and $d+1$ knots at $b > t_{m+d+1}$, in both τ and t, to form new knot vectors τ' and t' respectively. This adds $d+1$ new B-splines to each 'end' of $\mathcal{S}_\tau$ and $\mathcal{S}_t$, forming $\mathcal{S}_{\tau'}$ and $\mathcal{S}_{t'}$. Then use the fact that any $f = \sum_i c_i B_{i,d,\tau} \in \mathcal{S}_\tau$ can be represented in $\mathcal{S}_{\tau'}$ by adding $d+1$ zero coefficients to both ends of its coefficient vector, and similarly for $\mathcal{S}_t$. Use this to show that any $f \in \mathcal{S}_\tau$ belongs to $\mathcal{S}_t$.

3. Problem 4.6.

4. Problem 4.7. Hint: Use the fact that $\sum_j B_{j,d,\tau} = \sum_i B_{i,d,t} = 1$ and properties of discrete B-splines.

5. Problem 4.8. Hint: Algorithm 4.11 is very similar to algorithm 2.16, which you implemented in Assignment 2. Do not use the B-spline matrices R_k. The plots should look like Fig. 4.5 in the compendium (include control polygons).

6. Problem 4.10. Hint: Use linearity of the polar form / blossom.