Suggested problems week 1

1. (1) Prove that if A is a ring such that $1 = 0$, then A is the 0 ring.
2. (2) Prove Proposition 1.1. in [AM].
3. (3) Use the relation between ideals in $\mathbb{Z}$ and $\mathbb{Z}/(n) = \mathbb{Z}_n$ to show that the number of ideals in $\mathbb{Z}_n$ equals the number of positive integers dividing n.
4. (4) Let k be a field, and prove that the ideal
$$(x, y) = \{g_1 x + g_2 y \mid g_1, g_2 \in k[x, y]\} \subseteq k[x, y]$$
is not a principal ideal.
5. (5) Let $\mathfrak{a} \subset \mathbb{Z}[x]$ be the set of polynomials such that $f \in \mathfrak{a}$ if and only if $f(0)$ is even. Show that $\mathfrak{a}$ is an ideal, and that it is not a principal ideal.
6. (6) Prove that a ring A is an integral domain if and only if $A[x]$ is an integral domain.
7. (7) Convince yourself that for any ring A, we have an isomorphism $(A[x])[y] \cong A[x, y]$
8. (8) Let A be a ring. Prove that there exists some field k such that there is a surjective homomorphism $\phi\colon A \to k$.
9. (9) Let A be a ring. Prove that there exists a unique homomorphism $\phi\colon \mathbb{Z} \to A$.
10. (10) Let A be a ring, and let $a \in A$. Prove that there exists a unique homomorphism $\phi\colon \mathbb{Z}[x] \to A$ such that $\phi(x) = a$. Use this to describe the set of all homomorphisms $\phi\colon \mathbb{Z}[x] \to A$.

From Atiyah–Macdonald chapter 1: 1, 7, 8, 9, 10, 12, 15, 16.