MAT4210—Algebraic geometry I: Mandatory assignment.

14th February 2018

Problem 0.1

a) Let a and b be two natural numbers and deine an ideal in $k[x, y]$ by $\mathfrak{a} = (x^b y, y^a x)$. Determine a primary decompoition of $\mathfrak{a}$.

b) Determine the set $Z_+(\mathfrak{a})$ in $\mathbb{P}^1$. ★

Problem 0.2

a) Let $X \subseteq \mathbb{P}^3$ be the surface $X = Z_+(xw - yz)$. Show that W is irreducible.

b) For any elements α and β in the ground field k not both zero, let $L_{\alpha,\beta} = Z_+(\alpha x + \beta z, \alpha y + \beta w)$ and $M_{\alpha,\beta} = Z_+(\alpha x + \beta y, \alpha z + \beta w)$. Show that for all α and β the varieties $L_{\alpha,\beta}$ and $M_{\alpha,\beta}$ are lines in $\mathbb{P}^3$ lying on the surface X.

c) Show that $L_{\alpha,\beta} \cap L_{\alpha',\beta'} = \emptyset$ whenever $(\alpha; \beta) \neq (\alpha'; \beta')$ (as points in $\mathbb{P}^1$). Show that $L_{\alpha,\beta}$ and $M_{\alpha',\beta'}$ meet in one point. ★