MAT4215 - SYLLABUS 2018

The following is a list of the most important definitions and results in the course, in the order they appear in the Lecture notes. The points marked with * are especially important.

Chapter 1

Presheaves, Sheaves, sheaf saturation,
*Stalks and germs,
When a map of sheaves is injective/surjective
ker, im and coker sheaves, quotient sheaves
Examples where im/coker fails to be a sheaf
*Left exactness of Γ, and failure of its right exactness.
*$\mathcal{B}$-sheaves.
*Sheafification and its universal property.
Pushforward and inverse image of a sheaf. Adjoint properties.

Chapter 2

The Zariski topology on Spec$A$
$V(I)$ and $D(f)$
*Maps between rings vs morphisms of Spectra.
Spec of a DVR
*Definition of the structure sheaf on Spec$A$.
Definition of a scheme and morphisms of schemes
*The category of affine schemes vs the category of commutative rings

Chapter 3

Gluing of sheaves
Gluing of schemes
Gluing morphisms of schemes
*Maps from schemes into an affine scheme (Theorem 3.6)

Chapter 4

$\mathbf{P}^n$
A non-affine scheme
Affine line with doubled origin
Blow-up

Chapter 5

Connectedness, Irreducible, Reduced, Integral, Noetherian, quasi-compact, Finite type, Finite morphisms
The dimension of a scheme.

Chapter 6

*Definition of Proj(R) as a scheme.
*Maps between Proj's
The Veronese embedding is an isomorphism

Chapter 7

*Definition of fiber product for schemes
*Existence of fiber product for affine schemes. Superficial knowledge about the construction of the fiber product in general.
Scheme theoretic fiber

Chapter 8

The various constructions for $\mathscr{O}_X$-modules: sum, tensor product, Hom, ker, ..
Modules over Spec DVR.
Direct and inverse images of $\mathscr{O}_X$-modules
*The $\sim$ functor and its properties
*Quasi-coherent sheaves
Coherent sheaves
*Quasi-coherent sheaves on affine schemes
Proposition 8.17 (about the categories $QCoh$ and Mod) and the sketch of its proof.
Functorial properties of Quasi-coherence (f^* and f_*, ..)
*Closed immersions vs quasi coherent sheaves of ideals (only a sketch of the proof)

Chapter 9

Locally free sheaves.
*Invertible sheaves and Pic(X).

Chapter 10

The graded $\sim$ functor and its properties
*$\mathscr{O}(1)$. Sections of $\mathscr{O}(m)$ correspond to elements of R_m.
The associated graded module of a sheaf.
*The relation between graded modules on R and quasi-coherent sheaves on $Proj(R)$.
The correspondence between closed subschemes of $Proj(R)$ and saturated ideals of R.

Chapter 12

*Cech cohomology
*Cohomology of quasi-coherent sheaves on Spec$A$ for A noetherian
*Cohomology of $\mathscr{O}(m)$ on $\mathbb{P}^n$.
Euler characteristic, arithmetic genus
*Basic examples of using cohomology to get geometric information (e.g., plane curves, hyperelliptic curves..)