Mandatory assignment MAT4215 – Spring 2018

The assignment must be handed in by email or at the reception at Ullevål before 14:30, **Friday May 18th**. You need to solve at least two problems to pass. If you have any questions or comments about the problems, feel free to ask me in the lectures or email me at `johnco@math.uio.no`.

Problem 1. a) Give an example of a scheme X, a field K, and a morphism of ringed spaces $\operatorname{Spec} K \to X$ which is *not* a morphism of schemes.

b) Give an example of a graded ring A so that $\operatorname{Proj} A$ is an affine scheme.

Problem 2. Describe the scheme theoretic fibers in all points of the following morphisms.

(a) $\operatorname{Spec} \mathbb{C}[x,y]/(xy-1) \to \operatorname{Spec} \mathbb{C}[x]$
(b) $\operatorname{Spec} \mathbb{C}[x,y]/(x^2-y^2) \to \operatorname{Spec} \mathbb{C}[x]$
(c) $\operatorname{Spec} \mathbb{C}[x,y]/(xy) \to \operatorname{Spec} \mathbb{C}[x]$
(d) $\operatorname{Spec} \mathbb{Z}[x,y]/(xy^2-m) \to \operatorname{Spec} \mathbb{Z}$, where m is a non-zero integer.

Which fibers are irreducible or reduced?

Problem 3. Let k be a field; $A = k[x,y]$ and $I = (x,y)$. Let t be a formal variable and consider the ring

$$R = \bigoplus_{m \geq 0} I^m t^m \subset A[t]$$

(Here $I^0 = A$). So in this ring $\deg t = 1$, and $x, y \in A$ have degree 0.
(i) Show that we have a morphism

$$\pi : \operatorname{Proj} R \to \operatorname{Spec} A = \mathbb{A}^2_k$$

(ii) Show that $\operatorname{Proj} R$ coincides with the blow-up of $\mathbb{A}^2$ at the origin, and that π corresponds to the blow-up morphism.

[Hint: First find a simpler description of R. $\operatorname{Proj} R$ can be covered by two affine open sets; show that this gluing coincides with the one for the blow-up.]