MAT4215 - syllabus 2021

A word about preparing for the exam: The exam will be oral, so you should focus on concepts, definitions, examples and counterexamples in your preparation. You may also be asked for some basic computations, but nothing very complicated. Think about how the topics fit together, and why terms are defined in the way they are.
The following is a list of the most important definitions and results in the course, in the order they appear in the lecture notes. The points marked with * are especially important.

Chapter 1

Presheaves, Sheaves, morphisms between these
sheaf saturation
*Stalks and germs,
When a map of sheaves is injective/surjective
ker, im and coker sheaves, quotient sheaves
Examples where im/coker fails to be a sheaf
*Left exactness of Γ, and failure of its right exactness.
*Sheafification and its universal property.
Pushforward and inverse image of a sheaf. Adjoint properties of these.
* Sheaves defined on a basis.

Chapter 2

*The Zariski topology on Spec$A$,
*$V(I)$ and $D(f)$, properties of these
*Maps between rings vs morphisms of Spectra.
When is Spec A irreducible/connected
Main examples: Spec $\mathbb{Z}$, DVR, polynomial rings over fields, quotient rings, localization, ..
Primary decomposition and Spec

Chapter 3

*Definition of the structure sheaf on Spec$A$.
* Definition of a scheme
Morphisms of schemes
*The category of affine schemes vs the category of commutative rings
Relative schemes
Finite, finite type, ..
Open immersions
* Closed immersions
Reduced schemes
* Integral schemes

Chapter 4

Gluing of sheaves
Gluing of schemes
Gluing morphisms of schemes
*Maps from schemes into an affine scheme
R-valued points
* Definition of a (pre)variety

Chapter 5

Main examples:
A non-affine scheme (global sections)
* $\mathbf{P}^1$ (sheaves, morphisms, ..)
affine line with doubled origin
projective space
* Blow-up
Hyperelliptic curves

Chapter 6

Noetherian schemes (spec A noetherian iff A noetherian)
* Dimension
Normal schemes, normalization morphism

Chapter 7

Definition of fiber product for schemes
*Existence of fiber product for affine schemes. Superficial knowledge about the construction of the fiber product in general.
* Scheme theoretic fibers

Chapter 8

Separated schemes
* Affine schemes are separated
Example of a non-separated scheme
* separated vs. affine intersections

Chapter 9

*Definition of Proj(R) as a scheme.
* Distinguished open sets
*Maps between Proj's
The Veronese embedding
Weighted projective spaces

Chapter 10

The various constructions for $\mathscr{O}_X$-modules: sum, tensor product, Hom, ker, ..
Modules over Spec DVR.
Pushforward and pullback images of $\mathscr{O}_X$-modules
*The $\sim$ functor and its many properties
*Quasi-coherent sheaves
Coherent sheaves
*Quasi-coherent sheaves on affine schemes
Quasicoherent sheaves on $\mathbf{P}^1$.
* The categories $QCoh$ vs Mod. An understanding of the proof
Functorial properties of Quasi-coherence (f^* and f_*, ..)
*Closed immersions vs quasi coherent sheaves of ideals (only a sketch of the proof)

Chapter 11

* Locally free sheaves
Projective modules; examples
Dual sheaves
*Invertible sheaves and Pic(X).
* Invertible sheaves on $\mathbf{P}^1$.

Chapter 12

The graded $\sim$ functor and its properties
*$\mathscr{O}(m)$. Sections of $\mathscr{O}(m)$ correspond to elements of R_m.
The associated graded module of a sheaf.
*The relation between graded modules on R and quasi-coherent sheaves on $Proj(R)$.
The correspondence between closed subschemes of $Proj(R)$ and saturated ideals of R.
Locally free sheaves on $\mathbf{P}^1$.
* Important examples and exact sequences of sheaves on projective space

Chapter 13

*Cech cohomology, main properties
Long exact sequence for quasi-coherent sheaves

Chapter 14

*Cohomology of quasi-coherent sheaves on Spec$A$ for A noetherian
*Cohomology of $\mathscr{O}(m)$ on $\mathbb{P}^n$.
Euler characteristic, arithmetic genus
* Extended examples of using cohomology to get geometric information (e.g., plane curves, twisted cubic, hyperelliptic curves, non-split locally free sheaves, ..)