

MAT4215 - Syllabus 2022

A word about preparing for the exam: The exam will be oral, so you should focus on concepts, definitions, examples and counterexamples in your preparation. You may also be asked for some basic computations, but nothing very complicated. Think about how the topics fit together, and why terms are defined in the way they are.

The following is a list of the most important definitions and results in the course, in the order they appear in the lecture notes. The points marked with * are especially important.

Chapter 1

Presheaves, Sheaves, morphisms between these
sheaf saturation

*Stalks and germs,

When a map of sheaves is injective/surjective

ker, im and coker sheaves, quotient sheaves

Examples where im/coker fails to be a sheaf

*Left exactness of Γ , and failure of its right exactness.

*Sheafification and its universal property.

Pushforward and inverse image of a sheaf. Adjoint properties of these.

* Sheaves defined on a basis.

Chapter 2

*The Zariski topology on $\text{Spec} A$,

* $V(I)$ and $D(f)$, properties of these

*Maps between rings vs morphisms of Spectra.

When is $\text{Spec} A$ irreducible/connected

Main examples: $\text{Spec} \mathbb{Z}$, DVR, polynomial rings over fields, quotient rings, localization, ..

Primary decomposition and Spec

Chapter 3

*Definition of the structure sheaf on $\text{Spec} A$.

* Definition of a scheme

Morphisms of schemes

*The category of affine schemes vs the category of commutative rings

Relative schemes

Finite, finite type, ..

Open immersions

* Closed immersions

Reduced schemes

* Integral schemes

Chapter 4

Gluing of sheaves

Gluing of schemes

Gluing morphisms of schemes

*Maps from schemes into an affine scheme

R-valued points

* Definition of a (pre)variety

Chapter 5

Main examples:

A non-affine scheme (global sections)

* \mathbf{P}^1 (sheaves, morphisms, ..)

affine line with doubled origin

projective space

* Blow-up

Hyperelliptic curves

Chapter 6

Noetherian schemes (spec A noetherian iff A noetherian)

* Dimension

Normal schemes, normalization morphism

Chapter 7

Definition of fiber product for schemes

*Existence of fiber product for affine schemes. Superficial knowledge about the construction of the fiber product in general.

* Scheme theoretic fibers

Chapter 8

Separated schemes

* Affine schemes are separated

Example of a non-separated scheme

* separated vs. affine intersections

Chapter 9

*Definition of Proj(R) as a scheme.

* Distinguished open sets

*Maps between Proj's

The Veronese embedding

Chapter 10

The various constructions for \mathcal{O}_X -modules: sum, tensor product, Hom, ker, ..

Modules over Spec DVR.

Pushforward and pullback images of \mathcal{O}_X -modules

*The \sim functor and its many properties

*Quasi-coherent sheaves

Coherent sheaves

*Quasi-coherent sheaves on affine schemes

Quasicohherent sheaves on \mathbf{P}^1 .

* The categories *QCoh* vs *Mod*. An understanding of the proof

Functorial properties of Quasi-coherence (f^* and f_* , ..)

*Closed immersions vs quasi coherent sheaves of ideals (only a sketch of the proof)

Chapter 11

* Locally free sheaves

Projective modules; examples

Dual sheaves

*Invertible sheaves and $\text{Pic}(X)$.

* Invertible sheaves on \mathbf{P}^1 .

Chapter 12

The graded \sim functor and its properties

* $\mathcal{O}(m)$. Sections of $\mathcal{O}(m)$ correspond to elements of R_m .

The associated graded module of a sheaf.

*The relation between graded modules on R and quasi-coherent sheaves on $Proj(R)$.

The correspondence between closed subschemes of $Proj(R)$ and saturated ideals of R .

Locally free sheaves on \mathbf{P}^1 .

* Important examples and exact sequences of sheaves on projective space

Chapter 13

*Cech cohomology, main properties

Long exact sequence for quasi-coherent sheaves

Chapter 14

*Cohomology of quasi-coherent sheaves on $Spec A$ for A noetherian

*Cohomology of $\mathcal{O}(m)$ on \mathbb{P}^n .

Euler characteristic, arithmetic genus

* Extended examples of using cohomology to get geometric information (e.g., plane curves, twisted cubic, hyperelliptic curves, non-split locally free sheaves, ..)