MAT4215 - Syllabus 2024

A word about preparing for the exam: The exam will be oral, so you should focus on concepts, definitions, examples and counterexamples in your preparation. You may also be asked for some basic computations, but nothing very complicated. Think about how the topics fit together, and why terms are defined in the way they are.
The following is a list of the most important definitions and results in the course, in the order they appear in the lecture notes. The points marked with * are especially important.

1

Affine varieties and projective varieties are.
Regular functions.

2

*The Zariski topology on Spec$A$,
*$V(I)$ and $D(f)$, properties of these
*Maps between rings vs morphisms of Spectra.
When is Spec A irreducible/connected
Main examples: Spec $\mathbb{Z}$, DVR, polynomial rings, quotient rings, localization, ..
Fibres

3

Presheaves, Sheaves, morphisms between these
*Stalks and germs,
Pushforward sheaf
* Sheaves defined on a basis.

4

*Definition of the structure sheaf on Spec$A$.
* Definition of a scheme
Morphisms of schemes
* The category of affine schemes vs the category of commutative rings
Relative schemes
Open immersions
* Closed immersions
R-valued points

5

Gluing of sheaves
Gluing of schemes
Gluing morphisms of schemes
*Maps from schemes into an affine scheme

6

A non-affine scheme (global sections)
* $\mathbf{P}^1$ (sheaves, morphisms, ..)
affine line with doubled origin
* projective space

* Blow-up

7

* Integral schemes
* Noetherian schemes (e.g., spec A noetherian iff A noetherian)
* Dimension
Finite, finite type, affine
Normal schemes, normalization morphism

8

*Definition of Proj(R) as a scheme.
* Distinguished open sets
*Maps between Proj's
The Veronese embedding

9

Definition of fiber product for schemes
* Existence of fiber product for affine schemes.
Superficial knowledge about the construction of the fiber product in general.
* Scheme theoretic fibers

10

Separated schemes
* Affine schemes are separated
Example of a non-separated scheme
* separated vs. affine intersections

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When a map of sheaves is injective/surjective
ker, im and coker sheaves, quotient sheaves
Examples where im/coker fails to be a sheaf
*Left exactness of Γ, and failure of its right exactness.
*Sheafification and its universal property.
Pushforward and pullback of a sheaf. (The affine case is enough for pullbacks)
The various constructions for $\mathscr{O}_X$-modules: sum, tensor product, Hom, ker, ..

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*The $\sim$ functor and its many properties
**Quasi-coherent sheaves
Coherent sheaves
*Quasi-coherent sheaves on affine schemes
Quasicoherent sheaves on $\mathbf{P}^1$.
* The categories $QCoh$ vs Mod. Some understanding of the proof
*Closed immersions vs quasi coherent sheaves of ideals (only a sketch of the proof)
*Invertible sheaves and Pic(X).
* Invertible sheaves on $\mathbf{P}^1$.

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The graded $\sim$ functor and its properties
*$\mathscr{O}(m)$. Sections of $\mathscr{O}(m)$ correspond to elements of R_m.
The associated graded module of a sheaf.
*The relation between graded modules on R and quasi-coherent sheaves on $Proj(R)$.

Superficial knowledge of the correspondence between closed subschemes of $Proj(R)$ and saturated ideals of R.
* Important examples and exact sequences of sheaves on projective space

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*Cech cohomology, main properties
Long exact sequence for quasi-coherent sheaves
*Cohomology of quasi-coherent sheaves on Spec$A$ for A noetherian
**Cohomology of $\mathscr{O}(m)$ on $\mathbb{P}^n$.
Euler characteristic, arithmetic genus
* Extended examples of using cohomology to get geometric information (e.g., plane curves, twisted cubic, hyperelliptic curves, ..)