

This is the fourth set of exercises, based on the material in Chapter III, sections 2 and 3 in Silverman's book.

(1) Exercise 3.9

(2) Exercise 3.23

Note: Exercise 3.9 seems a little hard to do from scratch. Instead of doing (b) as stated, use instead the fact from Algebraic Geometry that the determinant of the Hessian matrix vanishes at a point  $P \in E$  if and only if the tangent line  $T_P$  of  $E$  at  $P$  intersects  $E$  only at  $P$ . (In more fancy language one might say that  $T_P$  has triple contact with  $E$  at  $P$ .) Then, at (c), prove that  $E[3]$  consists of *at most* 9 points. (Hint: use Bezout's theorem for curves in the projective plane.)