Overview of the material covered in the course MAT4230

1 Algebraic geometry over $\mathbb{C}$

The GAGA theorem
Chow's theorem

2 Differentials

Construction of the cotangent sheaf Ω_X via the module of differentials; relation to the diagonal
Ω_X^p, and T_X.
The (co)normal bundle and sequences.
$\Omega_{\mathbb{P}^n}$, Euler seqence
$P_n = H^0(X, nK_X)$ are birational invariants
Serre duality statement
The canonical bundle of a smooth divisor

3 Curves

Riemann-Roch theorem
Degree of K_X.
Hurwitz theorem, ramification index
Automorphism of curves (finiteness+Hurwitz bound)
The canonical embedding theorem (non-hyperelliptic curves)
Any curve can be projected isomorphically to $\mathbb{P}^3$.
Classification of curves of genus 0,1,2,3,4.
Rational normal curves
Clifford's theorem (not the proof)
Classification of curves in $\mathbb{P}^3$; the Castelnuovo bound
Elliptic curves as double covers and plan curves
j-invariant
Any elliptic curve over $\mathbb{C}$ is a torus

4 Flat families

Definition of flatness
Examples and non-examples of flat morphisms
Intuition about flat families
Flatness and fiber dimension
The flatness criterion (for curves)
Flat limits
Flatness and Hilbert polynomials
The twisted cubic degeneration
Smooth morphisms and $\Omega_{X|Y}$
Intuition for what smooth morphisms are for varieties in characteristic 0.

5 Grassmannian

Definition of $Gr(r, n)$; quotient construction, Plucker embedding.
The universal bundles, Grassmannian functor
Definition of $\mathbb{P}(\mathscr{F})$ for an $\mathcal{O}_X$-algebra $\mathscr{F}$; main examples ($\mathbb{P}(\mathscr{E})$, blow-ups).

6 Moduli and Hilbert schemes

Intuition for what moduli spaces are
Why some moduli functors fail to be representable
Examples of Hilbert schemes (the universal hypersurface)
Intuition for what the Hilbert scheme does
Not required: Existence of Hilb, tangent space of Hilb, construction of M_g via Hilb

7 Some concepts from differential geometry

Differential forms
De Rham cohomology groups
Poincare's lemma
Knowledge about defining cohomology using injective resolutions
Acyclic resolutions compute cohomology
De Rham's theorem
Stokes theorem
Poincare duality for $H^k(X, \mathbb{R})$.
Gysin map
Fundamental class and intersection numbers
Kunneth formula

8 Hodge theory

Orientation
Hodge star operator
Volume element
Inner product on p-forms
d^*, the adjoint of d.
Laplacians, harmonic forms
Intuition for the Hodge theorem (minimizing norm)
The sheaves $\mathscr{E}^{p,q}$
Dolbeault's theorem
The Lefschetz fixed point formula.
Application to automorphisms of Riemann surfaces
Hermitian metrics and Kahler forms
Kahler metrics
Examples: $\mathbb{C}^n, \mathbb{C}^g/\Lambda, \mathbb{P}^n$
The Hopf surface
The Hodge decomposition theorem for Kahler manifolds
Betti numbers of Kahler manifolds
The canonical Hodge decomposition (proof for Riemann surfaces)
The Hard Lefschetz theorem

9 Topology of algebraic varieties

First Chern class
The Lefschetz (1,1)-theorem
Statement of the Hodge conjecture

Statemenent of the Lefschetz hyperplane theorem (also for $\mathrm{Pic}(X)$).
Hodge numbers are deformation invariant
Some of the Hodge numbers are birational invariants
Computations: Hodge diamond of $\mathbb{P}^n$, hypersurfaces, products of two curves

10 Surfaces

The intersection pairing
Riemann Roch
Hodge index theorem
Nakai-Moishezon theorem
The main invariants: $\kappa(X)$, q, p_g.
Factorization of birational maps via blow-ups (statement only)
Examples of surfaces (rational, uniruled, elliptic, abelian, K3, Enriques)
Superficial knowledge about the Enriques-Kodaira classification of minimal complex surfaces