To complete the proof of PNT, let us show how Lemma 7.5 implies that  $\theta(x) \sim x$ . Assume that for some  $\lambda > 1$ , there exist arbitrarily large x with  $\theta(x) \geq \lambda x$ . Then, since  $\theta$  is nondecreasing, for each such x,

$$\int_{x}^{\lambda x} \frac{\theta(t) - t}{t^2} dt \ge \int_{x}^{\lambda x} \frac{\lambda x - t}{t^2} dt = \int_{1}^{\lambda} \frac{\lambda - t}{t^2} dt > 0,$$

which implies the divergence of  $\int_1^\infty [\theta(t)-t]/t^2 dt$ , contrary to Lemma 7.5. Similarly, if  $\theta(x) \leq \lambda x$  for some  $\lambda < 1$  and arbitrarily large x, we would have

$$\int_{\lambda x}^{x} \frac{\theta(t) - t}{t^2} dt \le \int_{\lambda x}^{x} \frac{\lambda x - t}{t^2} dt = \int_{\lambda}^{1} \frac{\lambda - t}{t^2} dt < 0,$$

which would again contradict the convergence of  $\int_1^\infty [\theta(t)-t]/t^2 dt$ . Thus

$$\lim_{x \to \infty} \theta(x)/x = 1,$$

and the proof is done.

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