MAT4250 EXERCISE SHEET 3

1. Cyclotomic fields

Exercise 1. Exercise 3, §10 in Neukirch.

(This is a special case of the Kronecker–Weber theorem, which states than any finite abelian extension K of $\mathbf{Q}$ (i.e., $\mathrm{Gal}(K/\mathbf{Q})$ is abelian) is contained in a cyclotomic field.)

Exercise 2. Let $K = \mathbf{F}_q$ be a finite field.

(a) Prove that the group of units $\mathbf{F}_q^\times$ in $\mathbf{F}_q$ is cyclic.
(b) Show that for each $n \geq 1$, there is a unique field extension $L/\mathbf{F}_q$ of degree n, with Galois group $\mathrm{Gal}(L/\mathbf{F}_q) \cong \mathbf{Z}/n$. In particular, any finite extension of a finite field is cyclic and of the form $\mathbf{F}_{q^n}$.

2. Arakelov divisors

Exercise 3. Exercise 1 in the notes on the arithmetic Riemann–Roch theorem.

Exercise 4. Prove that the pushforward map on arithmetic Chow groups is well defined.