

MAT4250 MANDATORY ASSIGNMENT

Choose one of the following topics and prepare a presentation of it for the class at **Tuesday November 13**. The presentation should be between 20 and 30 minutes long.

You may also choose a topic yourself. Ideally, the choice function should be injective.

Please email me at haakoak@math.uio.no about your choice.

Topic 1 (*S*-integers). Explain the notion of *S*-integers. State and prove Dirichlet's unit theorem for *S*-integers.

Possible sources: [Neu99, §11] or [Mil17, Ch. 5].

Topic 2 (Modules over Dedekind domains). Recall the structure theorem for finitely generated modules over principal ideal domains. State and give a sketch of the proof of the analogous result for finitely generated modules over Dedekind domains.

Possible sources: [Mil17, Theorem 3.31].

Topic 3 (Eisenstein polynomials). Define Eisenstein polynomials, and prove Proposition 1.3.8 in the following notes:

<https://www.math.uni-bonn.de/people/tian/ANT.pdf>

Also discuss a sample application of the result, for instance Example 1.3.9 in the same notes.

Topic 4 (Fermat's last theorem for regular primes). State and sketch the proof of the *first case of Fermat's last theorem for regular primes*, [Mil17, Theorem 6.8].

Topic 5 (Chow groups). Formulate the statement of Milnor patching, and use it to show the exactness of the units–Pic sequence. See Theorem 2.7 and Theorem 3.10 of

<http://sites.math.rutgers.edu/~weibel/Kbook/Kbook.I.pdf>

Use the units–Pic sequence to compute the Picard group of a suitable nonnormal variety of your choosing.

REFERENCES

- [Mil17] James S. Milne. *Algebraic Number Theory (v3.07)*. Available at www.jmilne.org/math/. 2017.
- [Neu99] Jürgen Neukirch. *Algebraic number theory*. Vol. 322. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]. Translated from the 1992 German original and with a note by Norbert Schappacher, With a foreword by G. Harder. Springer-Verlag, Berlin, 1999, pp. xviii+571.