MAT4250 EXERCISE SHEET 10

1. Class fields and conductors

Exercise 1. Compute $C_{\mathfrak{m}}$ when

(a) $K = \mathbb{Q}(\sqrt{-5})$, $\mathfrak{m} = (2, 1 + \sqrt{-5})$;
(b) $K = \mathbb{Q}(\sqrt{3})$ and $\mathfrak{m} = \infty_1\infty_2$ is the product of the real places of K.

Exercise 2. Exercise 3.13, p. 160 in Milne's notes.

Exercise 3. Exercise 3.15, p. 160 in Milne's notes.

2. S-integers

Exercise 4. Let K be a number field and S a finite set of finite places of K. The *ring of S-integers of K* is

$$\mathcal{O}_K^S = \{x \in K : |x|_v \leq 1 \text{ for all } v \notin S\}.$$

Thus, for instance, if $K = \mathbb{Q}$ and $S = \{2, 3\}$ then $\mathcal{O}_K^S = \mathbb{Z}[\frac{1}{2}, \frac{1}{3}]$.

Show that

$$(\mathcal{O}_K^S)^\times \cong \mu(K) \oplus \mathbb{Z}^{r+s+|S|-1}.$$