

MAT4250 EXERCISE SHEET 11

1. IDÈLES

Exercise 1. Prove that \mathbb{I}_K is locally compact.

Exercise 2. Show that the idèle class group is Hausdorff.

Exercise 3. Show that the map $c: \mathbb{I}_K \rightarrow \mathbb{R}_{>0}$, $c((a_v)_v) = \prod_v |a_v|_v$, is continuous.

Exercise 4. Let C_K and C_K^1 denote respectively the idèle class group, and the norm 1 idèle class group of K . Show that

$$C_K \cong C_K^1 \times \mathbb{R}_{>0}$$

as topological groups.

Exercise 5.

(a) By definition, $\mathbb{I}_{\mathbb{Q}} = \mathbb{R}^{\times} \times \prod'_p \mathbb{Q}_p^{\times}$. Use that $\mathbb{Q}_p^{\times} \cong \mathbb{Z} \times \mathbb{Z}_p^{\times}$ to show that

$$\mathbb{I}_{\mathbb{Q}} \cong \{\pm 1\} \times \mathbb{R}_{>0} \times \bigoplus_p \mathbb{Z} \times \prod_p \mathbb{Z}_p^{\times}$$

as topological groups.

(b) Show that $\mathbb{Q}^{\times} \cong \{\pm 1\} \times \bigoplus_p \mathbb{Z}$ and conclude that

$$\mathbb{I}_{\mathbb{Q}} \cong \mathbb{Q}^{\times} \times \mathbb{R}_{>0} \times \widehat{\mathbb{Z}}^{\times}.$$

(c) Which extension of \mathbb{Q} has Galois group isomorphic to $C_{\mathbb{Q}}^1$?