MAT4270: CHECKLIST

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This is a check list for final exam. The important point is not to memorize the answers, but to be able to explain the strategy to solve it (but you should go through the calculations also).

Finite groups

- what are the irreducible representations of $\mathbb{Z}/n\mathbb{Z}$?
- what are the irreducible representations of S_3?
- S_3 acts on $V = \langle e_1, e_2, e_3 \rangle$ by permutation of basis. Describe the self intertwiners of V.

Lie algebras

Look at the subalgebras of $\mathfrak{sl}_3$:

$$\mathfrak{n}_+ = \{\text{strictly upper triangular matrices}\}, \quad \mathfrak{b}_+ = \{\text{upper triangular matrices}\}$$

- describe their basis.
- what are the derived series and lower central series for them?
- which is nilpotent, which is solvable but not nilpotent?

Simple Lie algebras

For each of $\mathfrak{g} = \mathfrak{sl}_3$ and $\mathfrak{so}_5$;

- describe one (convenient) Cartan subalgbra $\mathfrak{h}$ of $\mathfrak{g}$.
- what are the roots for $\mathfrak{h}$?
- give an example of invariant bilinear form on $\mathfrak{g}$.
- describe the bilinear form on $\mathfrak{h}^*$ induced by the invariant form.
- describe the weight lattice, and draw the roots.
- what is the Weyl group of $(\mathfrak{g}, \mathfrak{h})$, and how does it act on the weight lattice?
- give an example of order on roots, describe simple positive roots.
- explain that another choice is conjugate by action of Weyl group.
- how do you draw the Dynkin diagram?

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