**MAT4360 - Fall 2017 - Curriculum**

The easiest way to define the curriculum is to say that it consists of the material that were covered in class (lectures and exercises), with the following book as background literature:

[M] G.J. Murphy: *$C^*$-algebras and operator theory*, Academic Press, 1990.

Here is a more precise description of the topics covered during the lectures (with some tentative references to Murphy's book):

- More on compact operators on Hilbert spaces.

  [M]: cf. Theorems 2.4.1, 2.4.6 and 2.4.7

- Non-unital $C^*$-algebras.

  [M]: cf. Section 2.1, p. 38–41.

- More on positivity in $C^*$-algebras.

  [M]: cf. Section 2.2, except Theorem 2.2.6 and the following comment.

- Approximate units. Norm-closed ideals and quotient of $C^*$-algebras

  [M]: cf. Section 3.1, but not Theorem 3.1.7 and the rest of this section.

- Positive linear functionals and states on $C^*$-algebras. The Jordan decomposition theorem.

  [M]: cf. Section 3.3, but Theorem 3.3.9 only when $B$ is a norm-closed ideal.

- Representations of $C^*$-algebras. The GNS-construction and the Gelfand-Naimark theorem.

  [M]: cf. Section 3.4, and Theorems 5.1.1, 5.1.3 and 5.1.4.

- The strong and the weak operator topologies. Von Neumann algebras and the double commutant theorem.

  [M]: cf. Section 4.1: from Lemma 4.1.4 to Example 4.1.2, and
  from Theorem 4.1.10 to Theorem 4.1.12;
  Section 4.2: from Theorem 4.2.5 to Corollary 4.2.8.

- The Kaplansky density theorem.

  [M]: cf. Theorem 4.3.3, but not part (4).

- Irreducible representations and pure states. The Gelfand-Raikov theorem.

  [M]: cf. Section 5.1: Theorem 5.1.2, and everything from Thm 5.1.5 to Thm 5.1.13.

- Irreducible representations and compact operators. Finite dimensional $C^*$-algebras.

  [M]: cf. Example 5.1.1 and the following comment, Theorem 2.4.9 (only when $A$ consists of compact operators), Remark 6.2.1 and Theorem 6.3.8.

- Algebraic versus topological irreducibility. The Kadison transitivity theorem.

  [M]: cf. Section 5.2. [NB: This section is not relevant for the oral examination.]