MAT4410 (2020 AUTUMN) EXERCISE SET 1

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Problem 1. Fix $a > 0$, and let $f(t)$ be an integrable function on $0 < t < a$. Show that

$$g(x) = \int_x^a \frac{f(t)}{t} dt$$

is also integrable on $0 < x < a$, and we have $\int_0^a f(t)dt = \int_0^a g(x)dx$.

Hint: write $g(x)$ as integrals on some 'slices' of $E = \{(t, x) : x < t < a, x < a\} \subset \mathbb{R}^2$.

Problem 2. Show that the Gamma-function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

satisfies the functional equation

$$\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} = \int_0^1 t^{x-1}(1-t)^{y-1} dt$$

for $x, y > 0$ (the right hand side is called the Beta function $B(x, y)$).

Hint: the intermediate formulas in the manipulation can be found in some calculus book or online resources (try looking up 'Gamma function, Beta function' for example). You still want to identify how Fubini's theorem is used, and how to check its assumption.

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