MAT4410 (2020 AUTUMN) EXERCISE SET 2

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Problem 3. Take $X = \mathbb{N}$ with counting measure μ (as usual, we make any subset of $\mathbb{N}$ measurable), so that $\ell^p(\mathbb{N}) = \ell^p(\mathbb{N}, \mu)$ is the space of p-summable sequences. For a given $n \in \mathbb{N}$, consider the function δ_n on $\mathbb{N}$ by setting

$$\delta_n(n) = 1, \qquad \delta_n(i) = 0 \quad \text{(otherwise)}.$$

(1) Let ϕ be a bounded linear functional on $\ell^1(\mathbb{N})$. Check that sequence $\phi(\delta_1), \phi(\delta_2), \ldots$ is bounded.
(2) What does it imply for the map $\ell^\infty(\mathbb{N}) \to \ell^1(\mathbb{N})^*$?
(3) Carry out the same consideration for $\ell^p(\mathbb{N})^*$ with $1 < p < \infty$.

Problem 4. Consider the unit circle $\mathbb{T} = \{e^{2\pi i t} \mid t \in \mathbb{R}\}$. We identify functions on $\mathbb{T}$ with the periodic functions in the t variable such that $f(t) = f(t+1)$, and consider the measure normalized 'angle measure' μ on $\mathbb{T}$ corresponding to the Lebesgue measure for the t variable, say, on $0 \leq t < 1$. Let $K(s,t)$ be a continuous function on $\mathbb{T}$. Consider the operation

$$(T_K f)(s) = \int K(s,t) f(t) d\mu(t).$$

(1) Show that T_K makes sense as a bounded operator on $L^1(\mathbb{T}, \mu)$. What is the norm of this operator?
(2) How about $L^p(\mathbb{T}, \mu)$?

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