10/10/2023

About assignment

Problem 1, (b) show that μ_1, μ_2 have the properties characterizing Lebesgue decomp. Also need the equality $\mu = \mu_1 + \mu_2$, do it on sets from $\mathcal{S}$, semi-algebra gen. $\mathcal{B}$.

Problem 2: Application of Tonelli's and Fubini's thm's.

Problem 3 (a) About positivity preserving for $\bar{\Phi}$, what is meant:

$f \geq 0$ η-a.e. on $X \longrightarrow \bar{\Phi}(f) \geq 0$ μ-a.e. on X.

$\left(\int_G \bar{\Phi}(f)\, d\mu \geq 0, \ \forall G \in \mathcal{G} \right)$.

For (1), show $\forall G \in \mathcal{G}$

$$0 \leq \int_G |\bar{\Phi}(f)|\, d\mu \leq \int_G \bar{\Phi}(|f|)\, d\mu.$$

|| hint using $\operatorname{sign}(\overline{\Phi(f)}) = \rho$,

$\int_G \Phi(f) \cdot \rho \, d\mu$, $\rho = \lim \rho_n$ pointwise, $|\rho_n| \leq 1$

$= \dots \int_G f \rho \, d\eta \dots \leq \int_G |f| d\eta$. $|\rho| \leq 1$.

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