MAT4450: CHECKLIST

Locally convex spaces

- How can you detect the difference of weak and strong topology on the Hilbert space $\ell_2\mathbb{N}$?
- Explain the weak*-compactness of the unit ball in a dual Banach space $\mathscr{X}^*$ (here $\mathscr{X}$ is a Banach space).
- Let $\mathcal{P}([0,1])$ be the set of regular Borel probability measures on $[0,1]$. Explain why / how the point masses δ_t for $0 \leq t \leq 1$ span $\mathcal{P}([0,1])$ up to convex span and closure (in which topology?).

Spectrum of operators

- Explain spectrum for matrices.
- Identify the spectrum of unilateral / bilateral shift operators.
- Explain the relation between spectrum and selfadjointness / positivity of operators.

Commutative Banach algebras

- Explain the ingredients of the Gelfand transform.
- Explain the relation between the C*-algebra generated by bilateral shift and the algebra of continuous function on $\mathbb{T}$.

Compact operators

- Give an example of compact operator, which is not of finite rank.
- Give an example of self adjoint compact operator, explain the diagonalizability.

Unbounded operators

- Give an example of symmetric operator for which deficiency indexes are not zero.
- Give an example of symmetric operator without selfadoint extension. Hint: kill a part of deficiency spaces from above example.
- Give an example of selfadjoint operator, describe its domain.

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