

Friedrichs

Step 4

$$A^{-1} \subset T'$$

$$u \in \mathcal{D}(T), v \in \mathcal{H}$$

$$\begin{aligned} (Tu, Av) &= (Tu, \underbrace{J^*v}_{\in \mathcal{K}}) = (u, J^*v)' \\ &= (Ju, v) = (u, v) \end{aligned}$$

$$\Rightarrow Av \in \mathcal{D}(T^*), \quad T^*Av = v$$

$$Av = J(J^*v) \in \mathcal{K}$$

$$\Rightarrow Av \in \mathcal{D}(T') \quad \text{so } \mathcal{D}(A^{-1}) = \text{Ran } A \subset \mathcal{D}(T')$$

But A^{-1} is s.o. ext. \Rightarrow max. symm.

T' is symm.

$$\begin{aligned} (T'u, v) &= (u, v)' = \overline{(v, u)'} = \overline{(T'v, u)} \\ &= (u, T'v) \end{aligned}$$