

## MAT4450 - Spring 2024 - Overview of the curriculum

The curriculum consists of the material that was covered during the lectures (and the given exercise sets), see the notes available online in the schedule for the course (and the solutions for the exercise sets).

The following topics were covered (with alternative references to Enstad's notes [Ens] and to Pedersen's book [Ped]):

- Topological preliminaries (Nets in topological spaces. Weak topologies. Compactness and nets. Tychonoff's theorem).  
[Ens] *Chapter 1*  
[Ped] *Sections 1.3, 1.4.3, 1.4.5, 1.4.6, 1.4.8, 1.5.2, 1.5.3, 1.5.4, 1.6.2, 1.6.10, 1.6.11*
- Quotient spaces [Ped] *Sections 2.1.5, 2.1.7*
- Adjoint operators [Ped] *Sections 2.3.9 – 2.3.12*
- Topological vector spaces with emphasis on locally convex vector spaces. The Hahn-Banach separation theorems. The weak topology on a normed space and the weak\* topology on its dual space.  
[Ens] *Sections 2.1–2.3* [Ped] *Section 2.4*
- Alaoglu's theorem. Compact convex sets. The Krein-Milman theorem.  
[Ens] *Sections 2.4–2.6* [Ped] *Sections 2.5.1–2.5.5, 2.5.7, 2.5.8*
- The Stone-Weierstrass theorem  
[Ens] *Sections 3.2* [Ped] *Section 4.3*
- Banach algebras.  
[Ens] *Section 3.1 and 3.3* [Ped] *Sections 4.1.1–4.1.8*
- The Gelfand transform on unital commutative Banach algebras.  
[Ens] *Sections 3.4 and 3.5* [Ped] *Section 4.2 (except 4.2.8 and 4.2.9)*
- C\*-algebras and the Gelfand transform on unital commutative C\*-algebras.  
[Ens] *Section 4.1* [Ped] *Sections 4.3.7 and 4.3.10–4.3.13*
- The continuous function calculus for normal elements in C\*-algebras, with applications to normal bounded operators.  
[Ens] *Section 4.2, 4.3 and 4.5* [Ped] *Sections 4.3.15, 4.3.16, 4.4.1–4.4.10*
- The Borel function calculus for normal bounded operators.  
[Ens] *Sections 4.6 and 4.7* [Ped] *Sections 4.5.4–4.5.8.*
- Partial isometries. Polar decomposition of bounded operators.  
[Ens] *Section 4.5* [Ped] *Sections 3.2.16–3.2.19*
- Fredholm operators.  
[Ens] *Section 5.2* [Ped] *Sections 3.3.10–3.3.19*
- The trace and trace class operators. (NB: without most of the proofs; the material on Hilbert-Schmidt operators is essentially known from previous courses).  
[Ens] *Section 5.1* [Ped] *Sections 3.4.1–3.4.8 and 3.4.12–3.4.13.*