MAT4460 - Autumn 2023
Further exercises

Exercises for November 1. From K. Strung's book, 7.3.1, 7.3.2, 7.3.3, 7.3.5, 7.3.6.

Exercises for November 8. From K. Strung's book, 8.7.1, 8.7.2, 8.7.3. Note that 8.7.2 is wrong as stated: The correct assumption on the product of matrix units in (i) should be

$$e_{ik}e_{lj} = \delta_{k,l}e_{ij} \text{ for } 1 \leq i, j, k, l \leq n,$$

and in part (ii) it should be

$$e_{ik}^{(r)}e_{lj}^{(s)} = \delta_{r,s}\delta_{k,l}e_{ij} \text{ for } 1 \leq i, j, k, l \leq n, 1 \leq r, s \leq m.$$

In the proof of the classification theorem for UHF (or Glimm) algebras, the following result is used (and is useful).

Lemma 0.1. *Let A be a unital C^*-algebra and p be a projection. If x is a self-adjoint element in A so that $0 \leq x \leq 1$ and $\|x - p\| < \varepsilon$ for some $\varepsilon > 0$, then $\|x^2 - x\| < 3\varepsilon$.*

The proof is immediate upon estimating the norm of $x^2 - x = (x^2 - px) + (px - p^2) + (p - x)$.

Exercises for November 15. From K. Strung's book, 9.6.1, 9.6.3, 9.6.5, 9.6.7, 9.6.8, 9.6.9.

Exercises for November 22. From K. Strung's book, 9.6.10, 9.6.11, 9.6.12.