About Proof of Lemma 9.5.7 in Strung's book. Some remarks.

Suppose X is a compact Hausdorff space and $\alpha: X \to X$ is a homeomorphism. Then (X,α) is said to be:

(a) minimal if every orbit is dense ($\forall x$, $\{\alpha^n(x) \mid n \in \mathbb{Z}\}$ is dense)

(b) topologically free if the set of aperiodic points $\mathrm{Per}^\infty(X,\alpha)$ is dense in X.

Here: $\mathrm{Per}^n(X,\alpha) = \{x \in X \mid \alpha^n(x) = x\}$ is the set of n-periodic points, for $n \in \mathbb{Z}$, ~~and~~ $\mathrm{Per}(X,\alpha)$ = the set of all periodic points (of some period n) and $\mathrm{Per}^\infty(X,\alpha)$ denotes the set of aperiodic points ($x \in X$ s.t. $\forall n \in \mathbb{Z}$, $\alpha^n(x) \neq x$).

Fact: If X is an infinite space, then minimality of (X,α) implies that (X,α) is top. free (at least assuming that the action α is effective: $n \in \mathbb{Z}$, $\sigma^n = id \Rightarrow n = 0$)

Fact: If (X,α) is top. free, suppose that V_0 is an open neighborhood of some point x_0. Let $N \geq 1$. Then there is an aperiodic point $y_0 \in V_0$. It follows that there is ~~a ne~~ an open set U (a neighbourhood of y_0) s.t.

$\alpha^n(U)$ for $-N \leq n \leq N$ become mutually disjoint sets.

(Since $\alpha(y_0) \neq y_0$ we can separate V_0 from $\alpha(V_0)$, possibly after shrinking V_0, and continue like this after applying α again)