UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in: MAT3500/MAT4500 — Topology

Day of examination: January 14th 2022

Examination hours: 9:00 – 13:00

This problem set consists of 2 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 (weight 30%)

Complete the following definitions, where X and Y are topological spaces.

(a) A *neighborhood* of a point $x \in X$ is ....
(b) A function $f: X \to Y$ is *continuous* if ....
(c) A (nonempty) space X is *connected* if ....
(d) A space X is *first-countable* if ....
(e) A space X is *normal* if ....
(f) An *m-dimensional manifold* X is ....

Problem 2 (weight 10%)

Give precise statements of the following results.

(a) The Lebesgue number lemma.
(b) The Tychonoff theorem.

Problem 3 (weight 10%)

Let A and B be compact subspaces of a Hausdorff space X.

(a) Prove that $A \cap B$ is compact in the subspace topology from X.

Problem 4 (weight 20%)

Let $p: E \to B$ be a covering map. Suppose that E is nonempty and path connected, and suppose that B is simply-connected.

(a) Prove that p is a bijection.
(b) Is p a homeomorphism? Justify your answer.

(Continued on page 2.)

Problem 5 (weight 30%)

Let X be a topological space. For each equivalence relation $R \subset X \times X$ let X/R be the set of R-equivalence classes in X, and let $q_R \colon X \to X/R$ be the canonical quotient map. We say that R is *good* if the quotient space X/R is Hausdorff.

(a) Is it true that each subspace of the product space

$$Y = \prod_{R \text{ is good}} X/R$$

is Hausdorff? Justify your answer.

Let $f \colon X \longrightarrow Y$ be the function with $\pi_R \circ f = q_R$ for each good R, where $\pi_R \colon Y \to X/R$ is the R-th projection mapping. Let $Z = f(X)$ be its image, with the subspace topology from Y.

(b) Prove that the corestricted function $g \colon X \to Z$, given by $g(x) = f(x)$ for each $x \in X$, is a continuous surjection with Hausdorff image.

(c) Prove that $g \colon X \to Z$ is a quotient map.

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