UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in: MAT3500/MAT4500 — Topology

Day of examination: December 8th 2021

Examination hours: 15:00 – 19:00

This problem set consists of 2 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 (weight 30%)

Complete the following definitions.

(a) A *basis* for a topology on a set X is a collection ....
(b) The *closure* of a subset A of a space X is ....
(c) A *homeomorphism* $f\colon X \to Y$ is ....
(d) A space X is *locally connected* if ....
(e) A space X is *compact* if ....
(f) A space X is *regular* if ....

Problem 2 (weight 10%)

Give precise statements of the following results.

(a) The Urysohn Lemma.
(b) The Urysohn Metrization Theorem.

Problem 3 (weight 30%)

Give $\mathbb{R}$ the usual metric topology, give $\mathbb{R}^4$ the product topology, and give

$$X = \{(a,b,c,d) \in \mathbb{R}^4 \mid ad - bc = 1\}.$$

the subspace topology. Consider the open subspaces

$$A = \{(a,b,c,d) \in X \mid a \neq 0\}$$
$$B = \{(a,b,c,d) \in X \mid b \neq 0\}.$$

(a) Show that A and B are both homeomorphic to open subsets of $\mathbb{R}^3$.
(b) Is X is a 3-dimensional manifold? Justify your answer.
(c) Is X compact? Justify your answer.

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Problem 4 (weight 10%)

Let $\mathbb{C} \cong \mathbb{R}^2$ have the usual metric topology, let $D^2 = \{z \in \mathbb{C} \mid \ |z| \leq 1\}$ be the unit disc, and let $S^1 = \{z \in \mathbb{C} \mid \ |z| = 1\}$ be the unit circle. Let

$$f \colon D^2 \longrightarrow \mathbb{C}$$

be a continuous function, and suppose that $f(z) = z$ for each $z \in S^1$.

(a) Show that $f(w) = 0$ for some $w \in D^2$.

Problem 5 (weight 20%)

Let $p \colon E \to B$ be a surjective quotient map, with B connected. Suppose for each $b \in B$ that $F_b = p^{-1}(b)$ is connected in the subspace topology from E.

(a) Show that if U, V is a separation of E then $p^{-1}(p(U)) = U$. Point out where you use that each F_b is connected.

(b) Show that E is connected. Point out where you use that p is a quotient map, that p is surjective, and that B is connected.

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