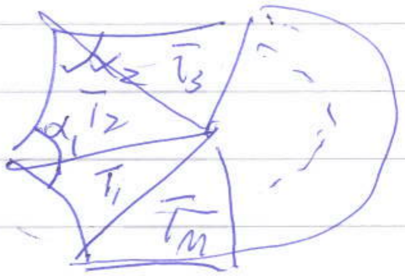
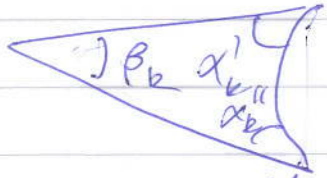
1

2.8.4 An n-gon is a convex set bounded by n segments. Then the n-gon has to lie inside every angle at the boundary (the vertices), and if we choose a point in the interior we can subdivide into triangles as in the picture to the right. Denote the angles in T_k by $\alpha_k', \alpha_k'', \beta_k$



as in



Then the area is $\sum_{k=1}^{n} (\pi - \beta_k - \alpha_k' - \alpha_k'')$

$$= n\pi - \sum_{k=1}^{n} \beta_k - \left(\sum_{k=1}^{n} (\alpha_k' + \alpha_k'')\right)$$

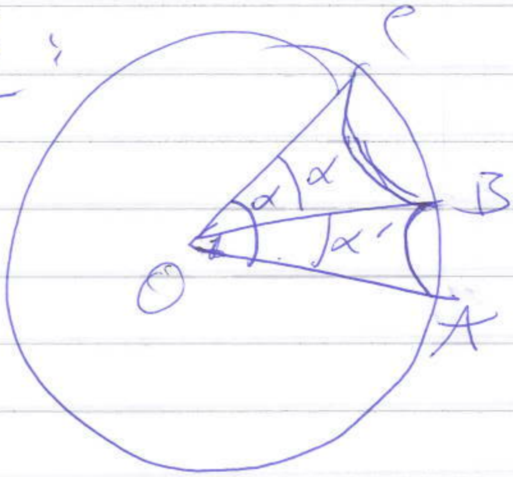
$$= n\pi - 2\pi - \sum_{k=1}^{n} \alpha_k$$

$$= (n-2)\pi - \sum_{k=1}^{n} \alpha_k$$

where α_k ~~is~~ $\alpha_k'' + \alpha_{k+1}'$ is the angle at the k'th vertex (set $\alpha_{n+1}' = \alpha_1'$)

(2.8.5 is a trivial observation of an important phenomenon)

2.8.6:



Denote the triangle by $T_{\alpha',\alpha''}$.

$\alpha'+\alpha''=\alpha$

a) Area $= \pi-\alpha'+\pi-\alpha'' = 2\pi-\alpha$
— depends only on α

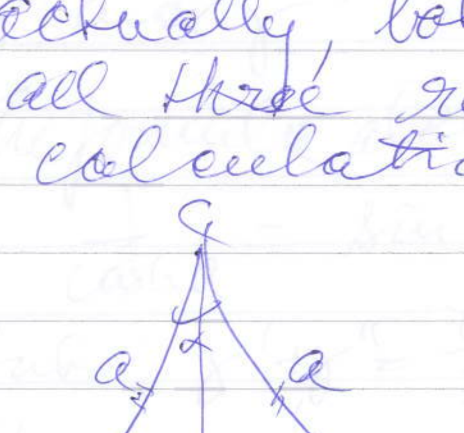
b) Consider two such triangles an assume $T_{\alpha',\alpha''} \cong T_{\beta',\beta''}$. Clearly $\alpha'+\alpha''=\beta'+\beta''$, so we may assume that the 4-gons are $\square OABC$ and $\square OAB'C$. If h is a congruence, we must have $h(O)=O$, and either $h(A)=A$ and $h(C)=C$ or $h(A)=C$ and $h(C)=A$. In the first case, $h=id$, so $B'=B$, and in the second h is a reflection in a diameter bisecting $\angle AOC$. Hence $\beta'=\alpha'$, $\beta''=\alpha''$ or $\beta'=\alpha''$, $\beta'=\alpha'$, so α alone does not determine $T_{\alpha',\alpha''}$.

2.9.4

Equal angles $\Rightarrow$ equal sides follows immediately from the second cosine formula.

Equal sides $\Rightarrow$ equal angles follow immediately from the first cosine formula.

(actually, both assertions follow from all three relations with a little calculation)



Let M be the midpoint of AB. Then $\triangle CBM \cong \triangle CAM$ by the congruence criterion C6 (SAS). Hence $\angle BMC = \frac{\pi}{2}$ and $\angle BCM = \frac{\alpha}{2}$

By the sine law we now have

$$\frac{\sin\frac{\alpha}{2}}{\sinh\frac{a}{2}} = \frac{\sin\frac{\pi}{2}}{\sinh a} = \frac{1}{2\sinh\frac{a}{2}\cosh\frac{a}{2}}$$

$$\Rightarrow \boxed{2\cosh\frac{a}{2}\sin\frac{\alpha}{2} = 1}$$