UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in: MAT4510 — Geometric structures

Day of examination: Wednesdy, December 7, 2016

Examination hours: 09.00 – 13.00

This problem set consists of 2 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

Let $\mathbb{H}$ be the upper half plane in $\mathbb{C}$.

a

Show that a fractional linear transformation

$$f(z) = \frac{az+b}{cz+d}$$

maps $\mathbb{H}$ to $\mathbb{H}$ if and only if $a, b, c, d \in \mathbb{R}$ and $ad - bc > 0$.

b

The above FLTs form the group $\text{Möb}^+(\mathbb{H})$ and they come in three different types; parabolic, hyperbolic and elliptic. Explain how these types are defined.

c

Determine the type of

$$f(z) = \frac{1+z}{1-z}$$

and show that f maps the imaginary axis to the unit circle.

d

The hyperbolic distance $d_{\mathbb{H}}(z, w)$ in $\mathbb{H}$ may be defined as follows. There is a unique $g \in \text{Möb}^+(\mathbb{H})$ such that $g(z) = i$ and $g(w) \in [i, \infty)$, i.e. $g(w) = ti$, where $t \geq 1$. Define $d_{\mathbb{H}}(z, w)$ to be $\ln |g(w)| = \ln t$. Let $w = e^{i\theta}$ with $\pi/2 \leq \theta < \pi$. Compute the hyperbolic distance from i to w.

(Continued on page 2.)

Problem 2

State and give an outline of the proof of the existence part of the classification theorem for compact, connected two-dimensional topological manifolds.

Problem 3

Consider the curve

$$\alpha(u) = (e^u, \int_0^u \sqrt{1 - e^{2t}}\, dt)$$

for $u < 0$, in the xz plane and let S be the corresponding surface of rotation around the z-axis.

a

Compute the the fundamental form of S.

b

Find a formula for the Gauss map from S. What is its image?

c

Compute the Gaussian curvature of S.

END