§ 1 Hilbert's axiom system

L1:5

Def The plane is a set S whose elements P are called points. Lines are subsets ℓ of S, $P \in \ell$ is the incidence relation which might or might not be satisfied.

Three sets of axioms: Incidence ax.

Betweenness

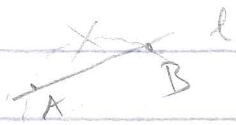
Congruence

+ axiom of continuity

+ axiom of parallels

Incidence axioms

I1: For every pair of distinct points A & B there is a unique line ℓ containing A & B.



I2: Every line contains at least two points

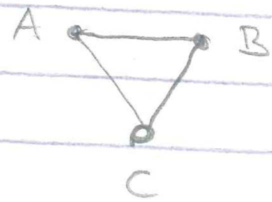
[$\Rightarrow$ can have $S = \{A\}$ & no lines, also $S = \{A, B\}$ w/ $\ell = \{A, B\}$ as only line]

I3: There are at least 3 points that do not lie on the same line.

⇝ Incidence geometry

Notation $\overline{AB}$ = unique line containing A & B

Ex 3 pts



⇝ triangle ABC

⇒ 3 lines

ExQ Give ex on incidence geometry w/ $\underset{5}{4}$ ~~elements,~~ points,

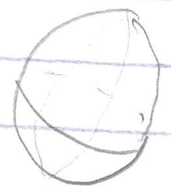
Ex $\mathbb{R}^2$ w/ lines given by sol. of $ax+by=c$

$\mathbb{Q}^2$ ——— || ———

$\mathbb{Z}^2$ ——— || ——— $a=y_1-y_2$, $b=x_1-x_2$

[Q: Is Check this!]

Non-ex S^2 sphere w/ lines = great circles



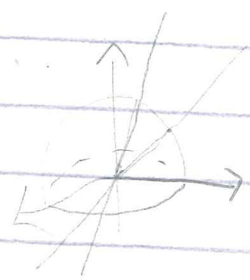
⇒ any pair of antipodal points lies

on ∞ many great circles ↯ I1

Ex $\mathbb{P}^2 = S^2/\sim$ $x \sim y \Leftrightarrow x, y$ antipodal

= the real projective plane

= {lines through O in $\mathbb{R}^3$}



great circle on S^2 = plane intersecting S^2

"Line" in $\mathbb{P}^2$ = plane through the origin in $\mathbb{R}^3$