PSS ① – Hints

1.3, 1.4 we solved on the blackboard

2.1.1 $\Phi(P)$: line through $(0,0,1)$ & $P \in S^2$ is given by

$$\ell(t) = (0,0,1) + t\,(P - (0,0,1)) .$$ For which t is $z=0$?

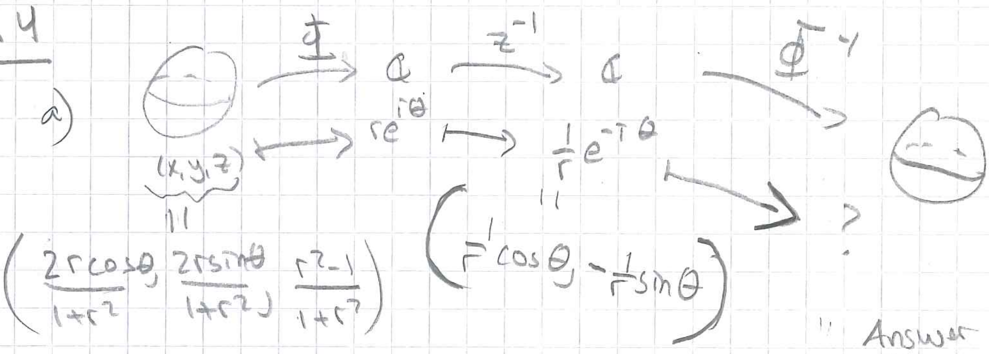
$\Phi^{-1}(Q)$: line through $(0,0,1)$ and $Q=(u,v,0)$:

$$\ell_Q = (0,0,1) + t\,((u,v,0)-(0,0,1))$$

For which t does this line intersect $S^2 = \{x^2+y^2+z^2=1\}$?

2.1.4

a)



" Answer $(x,y,z) \mapsto (x,-y,-z)$

π-rotation about x-axis.

b) $P=(x,y,z) \xrightarrow{P \mapsto -P} (-x,-y,-z)$

$$\downarrow \qquad\qquad\qquad \downarrow \Phi$$

$$\left(\frac{x}{1-z}, \frac{y}{1-z}\right) \qquad\qquad \left(\frac{-x}{1+z}, \frac{-y}{1+z}\right)$$

Write $x = r\cos\theta$, $y = r\sin\theta$ & use $r^2+z^2=1$

Answer $\quad z \longmapsto -\frac{1}{\bar{z}}$

2.2.2 We'll discuss it on PSS 2

2.2.3 Write down the matrices corresp to

$z \mapsto z+b$, $z \mapsto kz$, $k \in \mathbb{C} \setminus \{0\}$

$z \mapsto \frac{1}{z}$

and prove that any $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ w/ $ad-bc \neq 0$

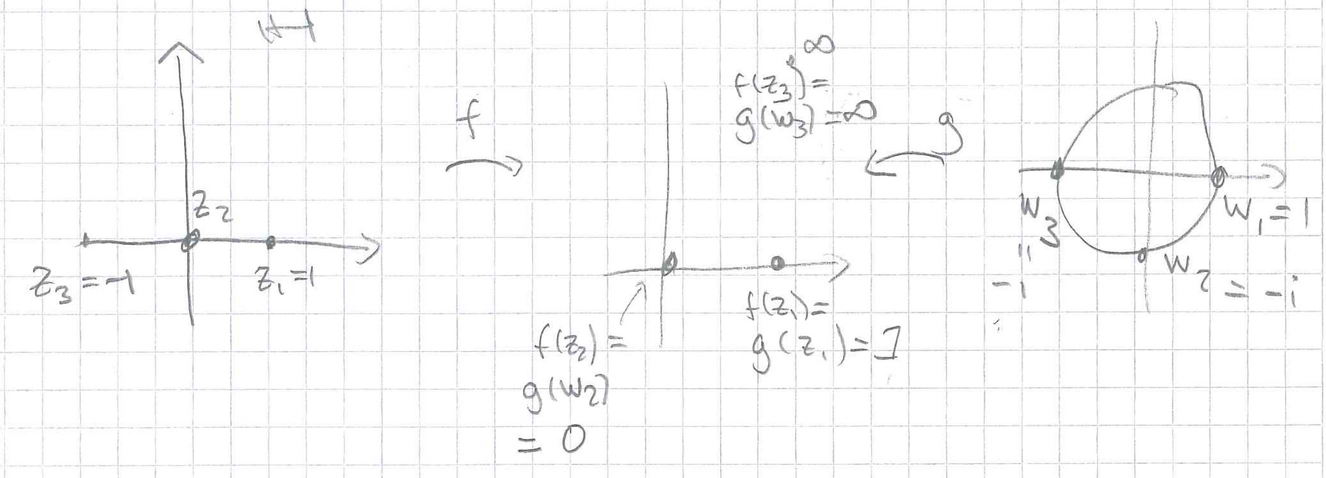
can be written as a product of such matrices.

[I needed 6 matrices in my product, maybe you found a simpler way?]

To prove Lma 2.2.1: Enough to prove Lma 2.2.1 for maps of type i) − iii).

i), ii) should be clear. For iii) use Exc. 2.1.4, + the fact that stereographic projection maps circles to $\bar{\mathbb{C}}$-circles & preserves angles.

2.2.5 For ex.



From lma 2.2.3 we get formulas for f & g.

Use that $g^{-1} = \frac{-dw+b}{cw-a}$ if $g = \frac{az+b}{cz+d}$

to compute $g^{-1} \circ f$. [In this case you should get $g^{-1} \circ f = \frac{iz+i}{z+i}$

2.2.7 a) Use Cor 2.2.5 ⇒ get

$f = \frac{az+b}{cz+d}$ w/ $a,b,c,d \in \mathbb{R}$.

2 cases: $ad-bc>0$ ⇒ ok

$ad-bc<0$ • Pre-compose w/ $\bar{z}$ & argue that we get an element in $Möb^-(\mathbb{H})$ which maps the first triple $\subset \bar{\mathbb{R}}$ to the second triple.

For the second statement:

Argue that a $\mathbb{H}$-line is uniquely det. by its 2 endpts in $\bar{\mathbb{R}}$

⇒ if we have 2 pairs of 2 lines w/ 1 common endpoint we get a triple $\subset \bar{\mathbb{R}}$

Use first part to get $f \in Möb(\mathbb{H})$ mapping triples to triples

If $f \in Möb^+(\mathbb{H})$ the result follows from the lecture, since know f maps $\mathbb{H}$-lines to $\mathbb{H}$-lines

If $f \in Möb^-(\mathbb{H})$: Use following: * $f = -\bar{g}$ for $g \in Möb^+(\mathbb{H})$

* $z \mapsto -\bar{z}$ maps $\mathbb{H}$-lines to $\mathbb{H}$-lines.

[argue why this is true].

b) The pf is in the book

c) Consider for ex the pts $(i, 2i)$ and $(i, 3i)$