PSS (2) – Hints 1

2.2.2 Use that f is an FLT + Lma 2.2.1

2.3.1 Sol a) hyperbolic and $f = h^{-1} \circ 2z \circ h$

w/ $h = \frac{2z-2}{2z-3}$

b) elliptic, $f = h^{-1} \circ g_\theta \circ h$ w/ $\theta = \frac{\pi}{3}$,

$h = \frac{2z-1}{\sqrt{3}}$

c) parabolic, $f = h^{-1} \circ (z-1) \circ h$ w/ $h(z) = -\frac{1}{z}$

Hint to find the h's: Solve $f(z) = z$ to find the fixpoints of f. Then use the method in the pf of Prp 2.3.2 to find h.

2.3.8 * For a), argue that you might assume that $C = \{|z| = 1\}$, and that the inversion is given by $\frac{1}{\bar{z}}$, and prove that a) holds in this case

(f.ex. if C has center m & radius r you can consider the FLT $h(z) = \frac{z-m}{r}$)

* For b), argue that this follows from the fact that $g(z) = -\bar{z}$ maps circles and lines intersecting the imaginary axis orthogonally to themselves (good idea to use Prp 2.3.3).

2.5.4 $\quad d_{\mathbb{H}}(z,-\bar{z}) = \left| \ln \frac{|z|+x}{|z|-x} \right|$

Use : The $\mathbb{H}$-line through z & $-\bar{z}$ has endpts $\pm r \in \mathbb{R}$

$$\Rightarrow d_{\mathbb{H}}(z,-\bar{z}) = \left| \ln\left(\frac{\bar{z}-r}{-\bar{z}+r} \frac{z+r}{z-r} \right) \right|$$ + simplifications of this expr.

(can be a good idea to use polar coord)

Distance from z to $i\mathbb{R}$ is by def

$$\inf_{y \in i\mathbb{R}} d_{\mathbb{H}}(z,y)$$

Clearly $d_{\mathbb{H}}(z, ir) = \frac{d(z,-\bar{z})}{2}$ by d4) + the fact

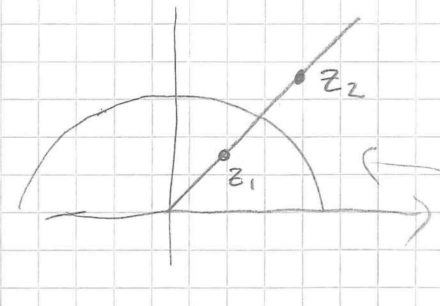
that $z \leftrightarrow -\bar{z}$ is an isometry which fixes $i\mathbb{R}$.

Can then use the hyperbolic law of sines to prove that

$d_{\mathbb{H}}(z, iy) \geq d_{\mathbb{H}}(z, i|z|) \quad \forall y \in \mathbb{R}$

Set of points having the same distance to $i\mathbb{R}_{>0}$:

are given by euclidean rays from the origin to ∞.



inversion in a special $\mathbb{H}$-line will do. Find it.

Idea of pf: Enough to show that 2 arb. pts z_1, z_2 on such a ray has the same distance to $i\mathbb{R}_{>0}$.
Do do that, find an suitable inversion which preserves $i\mathbb{R}_{>0}$ & maps z_1 to z_2

PSS 2 - Hints 2

2.6.1 Prove this in the case $l = i\mathbb{R}$ and argue that the general case follows from this

2.7.4 a) Prove that this holds for circles centered at O + use Möbius transf. in $\mathbb{D}$ to argue why it holds for a general hyperbolic circle

b) Map it to $\mathbb{D}$ & argue that the result then follows.