PSS 3 - Hints

2.8.1 Find a param. of the line so that

$$L = \int_0^1 \frac{|a_2 - a_1|}{b}\, dt = \frac{|a_2 - a_1|}{b}$$

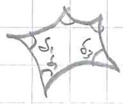
& use that

$$\cosh(L) = 1 + \frac{1}{2}L^2 + \frac{1}{4!} + \dots > 1 + \frac{L^2}{2}$$

2.8.4 Divide the n-gon into n triangles & use Proposition 2.8.5. You should get

$$A_{\mathbb{H}}(\text{polygon}) = \pi(n-2) - \sum_{i=1}^{n} \delta_i$$

where $\delta_1, \dots, \delta_n$ are the angles of the polygon



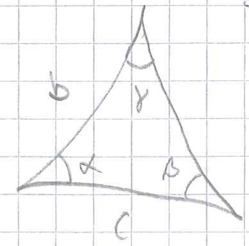
2.9.1 First use the hyperbolic law of sines to find a, then the two hyperbolic laws of cosine (to find first c & then γ (or other way around) (assuming the triangle is finite) (use cos, sinh, cosh injective on the intervals of interest)

2.9.2 Use the hyperbolic law of sines + area formula for triangles

$$\alpha = \beta \Rightarrow \sinh b = \sinh a \Rightarrow a = b \text{ since sinh inj}$$

$$a = b \Rightarrow \sin\alpha = \sin\beta \Rightarrow \alpha = \beta \text{ or } \underbrace{\alpha = \pi - \beta}_{\substack{\text{cannot happen since by assumption} \\ \gamma > 0 \Rightarrow \text{get contradiction from the area formula}}}$$

2.9.3 w.l.o.g. the greatest angle is γ w/ opposite side c. Enough to show $b < c$.



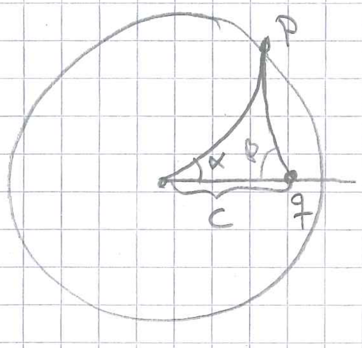
2 cases i) $\gamma < \pi/2 \Rightarrow \beta < \pi/2$

$\Rightarrow \sin\beta < \sin\gamma$

and the result follows from the hyperbolic sine rel

ii) $\gamma > \pi/2 \underset{\text{area}}{\overset{\pi}{\Longrightarrow}} \boxed{\beta < \pi - \gamma \ \& \ \beta < \pi/2} \Rightarrow \sin\beta < \sin\gamma$ also in this case

2.9.5 Use $\mathbb{D}$, w.l.o.g. the vertex w/ angle α at 0, vertex w/ angle β on pos $\mathbb{R}$-axis



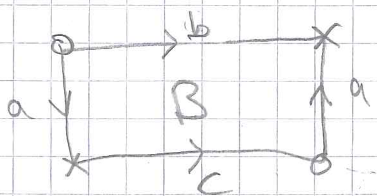
$\Rightarrow$ p determined by α,

q determined by c

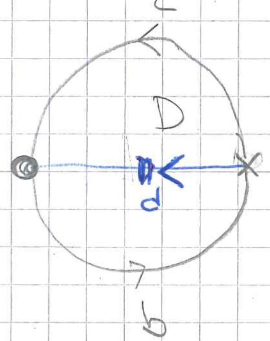
& β determined by p & q

3.1.1

(i)

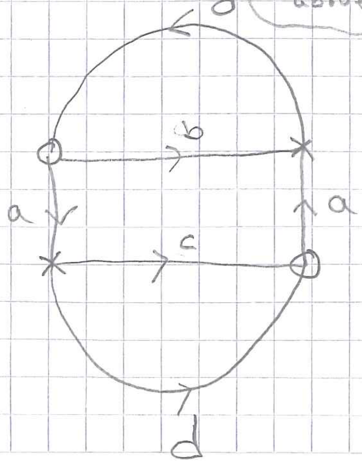


reason for dir. of b,c on B: when enter ○ below you should come out in ○ above

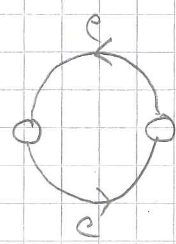


cut ↓ along D & identify sides b,c w/ sides on B

$\rightsquigarrow$



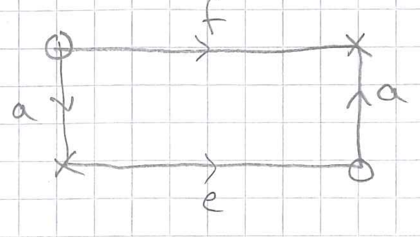
$= \lceil ad = e \rfloor =$

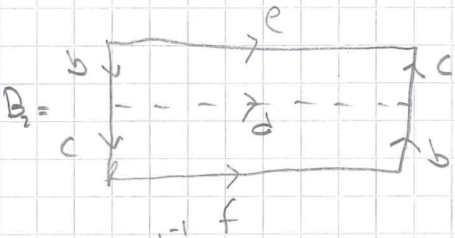


$= P^2$

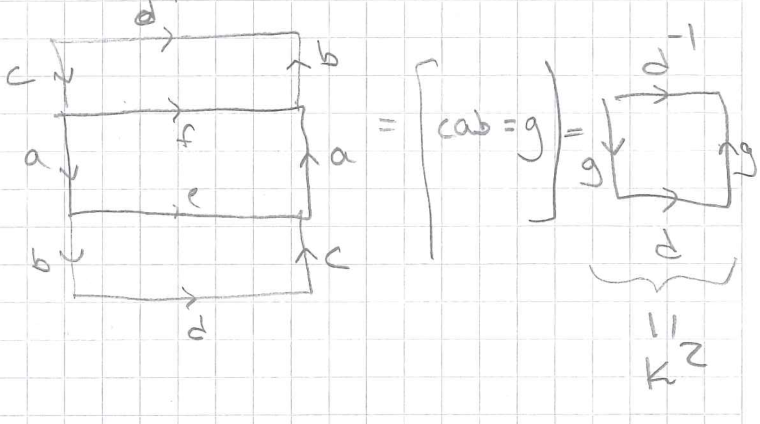
PSS ③ - Hints ②

(ii)

$B_1 =$ 

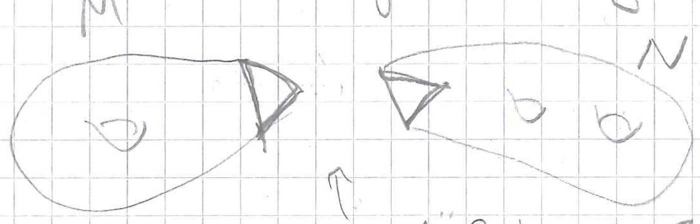
$B_2 =$ 

Glue along e, f



$= \left[cab = g \right] =$ ... $= K^2$

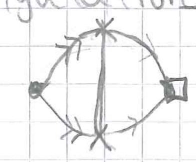
3.1.7 Use triangulation on $M \# N$ induced by the triangulation on M & N, where you glue them together along a triangle



get 2 triangles, 3 edges & 3 vertices less.

M irreducible $\iff M \simeq T^2, P^2$ or S^2 :

Find good triangulations for S^2, P^2, T^2 to compute χ.

ex $\chi(S^2)$:  $s = 2,\ e = 3,\ v = 3$ $(\bullet, \square, \times)$

$\Rightarrow \chi(S^2) = 2$

sim. $\chi(T^2) = 0,\ \chi(P^2) = 1$

If now $\chi(M) = \underbrace{\chi(N_1)}_{2-2m_1-n_1} + \underbrace{\chi(N_2)}_{2-2m_2-n_2} - 2$ ← Thm 3.1.4

we get $m_1 = m_2 = n_1 = n_2 = 0$ if $\chi(M) = 2$,

$n_1 = n_2 = 0$ if $M = T^2$ since T^2 orientable (also applies in $M = S^2$ case)

$\chi(T^2) = 0 \Rightarrow m_1 + m_2 = 1 \Rightarrow m_1 = 0, m_2 = 1$ or $m_1 = 1, m_2 = 0$

$M = P^2 \Rightarrow m_1 + m_2 = 0, \quad n_1 + n_2 = 1$

and from this you can draw the conclusion that S^2, P^2, T^2 are irreducible.

Remains: Argue that no other surface is irreducible.

3.4 Use 3.7 & $S(m,n) = \#_m T^2 \# \#_n P^2$

3.5 Use Lma 3.1.11 repeatedly to show

$D^2/abc^{-1}bdacd^{-1} \approx \#_3 P^2 = S(0,3) \approx S(1,1)$

$n > 0 \Rightarrow$ non-orientable

χ : $2 - 2 \cdot 1 - 1 = -1$

3.6 Use 3.7 + the fact that homeomorphic surfaces have the same Euler char $\Rightarrow \chi(M_2^4) = 2$

Now use Thm 3.1.14.

5.1.3 Assume $df = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{bmatrix}$ in this basis

$\Rightarrow df(x_u) = \alpha_1 y_u + \beta_1 y_v$

&

$df(x_v) = \alpha_2 y_u + \beta_2 y_v$

But $x_u = dx(1,0)$, $x_v = dx(0,1)$, $y_u = dy(1,0)$, $y_v = dy(0,1)$

$\Rightarrow J(y^{-1}fx)(1,0) \underset{\text{chain rule}}{=} dy^{-1}df\,dx(1,0) = dy^{-1}(\alpha_1 y_u + \beta_1 y_v)$

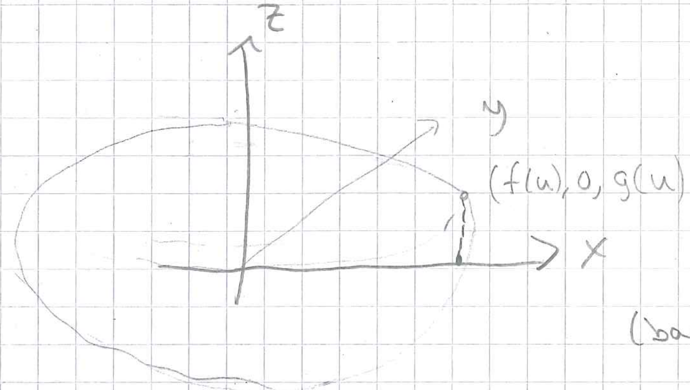
$= \alpha_1(1,0) + \beta_1(0,1)$

& sim for $J(y^{-1}fx)(0,1)$

PSS 3 - Hints 3

5.1.4 Sol $x(u,v) = (f(u)\cos v, f(u)\sin v, g(u))$

Hint: For each u, view $f(u)$ as a point in the xy-plane. This traces out a circle of radius $f(u)$ in the xy-plane when $(f(u), 0, g(u))$ is rotated around the z-axis



(bad picture)

Regular Check that $x_u = (f'(u)\cos v, f'(u)\sin v, g'(u))$
$x_v = (-f(u)\sin v, f(u)\cos v, 0)$

are lin indep at all pts,

Will get the condition

$f'(u)\, g'(u) \neq 0 \Rightarrow \alpha$ must be regular

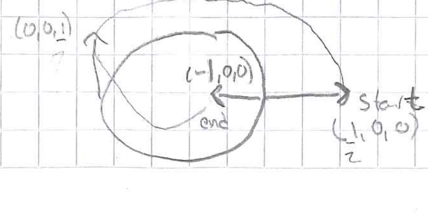
5.1.5 Might need to put length-restrictions on v

Regular $\alpha'(u) + v\delta'(u)$ & $\delta(u)$ must be lin. indep.

5.1.6 You can for example view it as a ruled surface w/ α given by the unit circle in $\mathbb{R}^2 \subset \mathbb{R}^3$, and δ a curve turning one of the normal vectors to S^1 "a half way around" as we traverses the circle, given by

$\left(\cos\frac{u}{2}\cos u, \cos\frac{u}{2}\sin u, \sin\frac{u}{2}\right)$

for example



5.2.1 Sim to the pf of Prp 5.2.3.