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MAT4510
Geometriske strukturer
Høst 2020

PREPARATION FOR ORAL EXAM

Book

- Jahren, Bjørn, *Geometric Structures in Dimension two*

Results that you should know

As a preparation for the oral exam, make sure that you know the following (an X indicates that you should be able to use it/prove it).

Result	**How to prove it**	**How to use it**	**Good exercises**
Lemma 2.2.1		X	2.2.2
Lemma 2.2.3	X	X	2.2.5
Corollary 2.2.7	X	X	
Proposition 2.2.9		X	
Proposition 2.3.2		X	2.3.1
Proposition 2.3.3		X	2.3.8
Lemma 2.4.2	X	X	2.6.1
Lemma 2.4.4	X	X	
Lemma 2.4.5	X	X	
Proposition 2.8.5	X	X	2.8.4
Proposition 2.9.1 - 2.9.4		X	2.9.1 – 2.9.3
Proposition 2.9.6	X	X	
Lemma 3.1.11		X	3.1.5
Lemma 3.1.13	X	X	
Theorem 3.1.14		X	3.1.6
Theorem 4.2.1		X	
Proposition 5.4.3	X	X	
Proposition 5.5.3		X	5.5.1
Theorema Egregium		X	
Proposition 5.5.6	X	X	5.5.10
Proposition 5.6.5	X	X	
Theorem 5.6.7	X	X	
Theorem 5.6.9		X	5.6.6
Proposition 5.6.11		X	
Theorem 5.6.12 – 5.6.14		X	
Lemma 5.7.2	X	X	
Theorem 5.7.6	X	X	
Theorem 5.8.1		X	
Theorem 5.9.2		X	5.9.6 – 5.9.8

Questions that you might get

Below are some examples of typical questions that you might get on the exam.

Axioms

- Which are the three most important axioms in Hilbert's axiom system for plane geometry, according to you? Motivate you answer (there are no wrong answers, as long as you can motivate it).

Hyperbolic space

- Describe the $\mathbb{D}$– and $\mathbb{H}$–model for the hyperbolic plane, including lines, congruences, betweenness, how to measure angles and distance (and how to use these definitions in some examples like exercises 2.5.4, 2.7.4, 2.8.1.)

Trigonometry

- Describe the following congruence criteria, and motivate which of them hold in euclidean and hyperbolic geometry, respectively.
  - SSS
  - AAA
  - ASA
  - SAS
  - SAA

Topological surfaces

- Describe S^2, P^2, K^2, T^2 as polygons with sides identified.
- Explain why P^2 and K^2 are not orientable, while S^2 and T^2 are.
- Solve Exercise 3.1.4
- Solve Exercise 3.1.7

Geometic structures on surfaces and the Gauss-Bonnet theorem

- Which closed (= compact without boundary) surfaces can be given an Euclidean structure, and which closed surfaces can not? Motivate!
- Which surfaces (= compact without boundary) can be given a hyperbolic structure, and which closed surfaces can not? Motivate!

Riemannian geometry

- Give the definition of a Riemannian surface + some examples.
- Give the definition of a (Riemannian) isometry + some examples.
- Give the definition of the following and tell if they are intrinsic or extrinsic. Motivate!
  - First fundamental form
  - Second fundamental form

  - Gaussian curvature
  - Area and arc-length
  - Covariant second derivative
  - Geodesic
  - Christoffel symbols
  - Geodesic curvature
- Give an example of a surface of rotation (also called a surface of revolution) and compute its first and second fundamental form, Gaussian curvature, Christoffel symbols, area, give examples of some geodesics. (See for example Exercise 5.1.4, 5.3.1)
- Give a definition of the exponential map and say something about its properties.
- Define geodesic normal coordinates, and derive the expression for the geodesic polar coordinates in Example 5.7.1 (iii).
- State the three most important consequences of the Gauss-Bonnet theorem, according to you. Motivate your choice (no wrong answer).

In addition, if you feel that you are comfortable with solving the recommended exercises for the problem solving sessions (see separate sheet) you should be very well prepared for the exam.