11th October, 2023

MAT4510

Mandatory assignment 1 of 1

Submission deadline

Thursday 26th October 2023, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) no later than the same day as the deadline.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

Problem 1. Let

$$S := \{(x, y, z) \in \mathbb{R}^3 \mid xyz = 1 \text{ and } x, y, z > 0\}.$$

(i) Show that S is a regular surface.

(ii) Calculate the Gauss curvature of S.

Problem 2.

Let a, b be real numbers with $0 < b < a$, and let S be the surface of revolution obtained by revolving the circle

$$\{(x, 0, z) \in \mathbb{R}^3 \mid (x-a)^2 + z^2 = b^2\}$$

about the z–axis. Describe the region in S where the Gauss curvature is positive and the region where it is negative.

Problem 3.

Let S be the surface of revolution obtained by revolving a curve $\gamma : I \to \mathbb{R}^3$ about the z–axis, where γ has the form

$$\gamma(t) = (r(t), 0, h(t))$$

and $r > 0$. Recall that S has a local parametrization $F : I \times J \to S$ given by

$$F(t, \phi) = (r(t)\cos\phi, r(t)\sin\phi, h(t))$$

for any open interval J of length 2π.

(i) For fixed $\phi \in J$ the curve $c : I \to S$ given by

$$c(t) := F(t, \phi)$$

is called a *line of longitude*. Show that c is a geodesic if and only if γ has constant speed.

(ii) For fixed $t \in I$ the curve $c : J \to S$ given by

$$c(\phi) := F(t, \phi)$$

is called a *line of latitude*. Show that c is a geodesic if and only if $\dot{r}(t) = 0$.

Problem 4.

Let S be a regular surface. A 1–*form* on S is a rule α that assigns to every point $p \in S$ a linear map $\alpha_p : T_pS \to \mathbb{R}$. A 1–form α is called *smooth* if for every smooth vector field X on S the map

$$\alpha(X) : S \to \mathbb{R}, \quad p \mapsto \alpha_p(X_p)$$

is smooth.

(i) Show that for any smooth 1–form α on S the map

$$d\alpha : \mathfrak{X}(S) \times \mathfrak{X}(S) \to C^\infty(S)$$

given by

$$d\alpha(X, Y) = \partial_X(\alpha(Y)) - \partial_Y(\alpha(X)) - \alpha([X, Y])$$

is $C^\infty(S)$–bilinear.

(ii) Deduce from (i) that for any $p \in S$ there is a unique skew-symmetric, bilinear map

$$(d\alpha)_p : T_pS \times T_pS \to \mathbb{R}$$

such that for any smooth vector fields X, Y defined in a neighbourhood of p in S one has

$$d\alpha(X, Y) = (d\alpha)_p(X_p, Y_p).$$