UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MAT4510 — Geometric Structures

Day of examination: Friday, December 12, 2014

Examination hours: 09.00 – 13.00

This problem set consists of 2 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

1a

Show that the elements in $\textit{Möb}^+(\mathbb{H})$ which map the imaginary axis to the $\mathbb{H}$–line $|z| = 1$ are precisely the ones that can be written as one of the types

$$(1)\quad f(z) = \frac{z-c}{z+c} \text{ where } c > 0\,, \text{ or } \quad (2)\quad f(z) = -\frac{z-c}{z+c} \text{ where } c < 0\,.$$

1b

Discuss how the classification of transformations of type (2) varies with c.

Which of these transformations have i as fixpoint? Write these on standard form.

1c

Find all possible *inversions* mapping the imaginary axis to $|z| = 1$. Which $\mathbb{H}$–lines are they inversions in?

Problem 2

2a

In a hyperbolic triangle with angles α, β, γ, let c be the (hyperbolic) length of the side opposite to the angle γ. Show that the second law of cosines can be written in the form

$$\cos(\alpha + \beta) - \cos(\pi - \gamma) = \sin\alpha \sin\beta(\cosh c - 1).$$

Use this to show that if α, β and γ are positive numbers, there exists a hyperbolic triangle vith vertex angles α, β, γ if and only if $\alpha + \beta + \gamma < \pi$.

Why is such a triangle unique up to congruence?

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2b

Show that there exists a regular hyperbolic hexagon (six edges and vertices) with vertex angle $\frac{\pi}{2}$.

Determine its area, the length of its edges and the radius of its circumscribed circle.

Problem 3

In this problem S is the *catenoid*, obtained by revolving the curve $y = \cosh x$ around the x-axis. If $a \in \mathbb{R}$, let $\mathcal{C}_a$ be the intersection between S and the plane $x = a$, and let S_a the region of S bounded by $\mathcal{C}_0$ and $\mathcal{C}_a$.

3a

Find a parametrization of S and compute its fundamental form.

Compute the area of S_a.

3b

Find a formula for the Gauss map from S and show that it is injective. What is its image?

3c

Compute the Gaussian curvature K of S.

Compute $\iint_{S_a} K\,dA$ and show that $\lim_{a\to\infty} \iint_{S_a} K\,dA$ is finite.

3d

Verify that $\mathcal{C}_0$ is a geodesic on S.

Explain why the geodesic curvature k_g^a of the curve $\mathcal{C}_a$ is constant and compute k_g^a.

Problem 4

Assume that a closed geodesic $\mathcal{C}$ cuts a surface of negative curvature into two connected components R_1 and R_2. Prove that none of these components can be homeomorphic to a disk or a Möbius band.

THE END