MAT 4520-2012
MANDATORY ASSIGNMENT

Deadline: *You must turn in your paper before Thursday, April 26., 2012, 2.30 p.m. in the specially designated box in the 7th floor. Remember to use the official front page available at*

http://www.mn.uio.no/math/studier/admin/obligatorisk-innlevering/obligforside-eng.pdf

If you due to illness or other circumstances want to extend the deadline, you must apply for an extension to Robin Bjørnetun Jacobsen (room B718, NHA, e-mail:studieinfo@math.uio.no, phone 22 85 59 07). Remember that illness has to be documented by a medical doctor! See

http://www.mn.uio.no/math/studier/admin/obligatorisk-innlevering/obligregelverk-eng.html

for more information about the rules for mandatory assignments

Instructions: *The assignment is compulsory, and students who do not get their paper accepted, will not get access to the final exam. Students who do not get their original paper accepted, but who have made serious and documented attempts to solve the problems, will get one chance of turning in an improved version.*

Problem 1

A map $\varphi : \mathbb{R}^3 \to \mathbb{R}^4$ is given by

$$\begin{aligned} x &= \nu \\ y &= \lambda \\ z &= \mu \\ w &= \lambda\nu - \mu\nu^2 + \nu^4, \end{aligned}$$

where λ, μ, ν denote coordinates in $\mathbb{R}^3$ and x, y, z, w denote coordinates in $\mathbb{R}^4$.

(a) Show that φ is a proper differentiable embedding.

Let $V = \varphi(\mathbb{R}^3)$ and let $f : V \to \mathbb{R}^3$ be the restriction of the projection $(x, y, z, w) \to (y, z, w)$.

(b) Determine the set $S \subset V$ where f is not an immersion and show that S is a closed C^∞ submanifold of V.

(c) Determine the set $T \subset S$ where $f|S$ is not an immersion.
Is T a C^∞ submanifold ? What about $f(T)$, is $f(T)$ a C^∞ submanifold of $\mathbb{R}^3$?

Problem 2

Let $X = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$ and $Y = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$ be vector fields in $\mathbb{R}^2$.

(a) Find the flow $\varphi(x, y, t)$ of X, and show that the flow of Y is given by $\psi(x, y, t) = (x\cos t - y\sin t, y\cos t + x\sin t)$.

(b) Calculate the Lie derivative $L_Y X$ by using the flow ψ and also by calculating $[Y, X]$ directly.

(c) Let $p = (x_0, y_0) \neq (0,0)$. Explain why there is a neighborhood U around p and a coordinate system defined on U such that X and Y become the coordinate vector fields of this coordinate system.

Spivak Chapter 7:

Problem 5, 6 and 21. (The form $d\theta$ in Problem 21 is the one defined in Problem 20).

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